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APPLICATION OF AN ITERATIVE GUIDANCE MODE TO A LUNAR LANDING

by Helmut J. Horn

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DEFINITION OF SYMBOLS

Symbol	Definition
t	time
T	time at target
τ	time to complete consumption
R	value to be extremized
x, y, $\dot{x}$, $\dot{y}$	horizontal and vertical coordinates and velocity components
ξ, η, $\dot{\xi}$, $\dot{\eta}$	rotated coordinates and velocity components
ϵ	angle of rotation of ξ against x
φ	thrust angle
θ	path tangent
Φ	central angle to point of origin
F	thrust
m, $\dot{m}$	mass and flow rate
g	constant of gravity
a, b, c, d, K_1, K_2	coefficients of the steering equation
a	acceleration
v	velocity
Subscript	
i : v_i	initial or instantaneous
g : v_g	caused by gravity
T : v_T	total or final
L : g_L	lunar

DEFINITION OF SYMBOLS (Cont'd)

Superscript	Definition
$\sim$: $\tilde{\varphi}$	preliminary value for restricted case
$*$: F^*	previous value

APPLICATION OF AN ITERATIVE GUIDANCE MODE TO A LUNAR LANDING*

SUMMARY

The purpose of a guidance system is to direct a vehicle from a given set of initial conditions to a predetermined set of final conditions with a minimum expenditure of propellants. Optimization of trajectories for minimum propellant consumption can be achieved with use of the calculus of variation. This method requires a large volume of numerical computation, making a real-time solution on board the vehicle impractical.

For highly simplified problems, e.g., for a homogeneous gravitation field, closed form solutions are available. These solutions provide an approximate answer for a real case. If applied repetitively, they can be used as steering equations. These approximations become stepwise better and better, and the end conditions should be reached with any desired degree of accuracy (disregarding instrument errors).

Application of this "Iterative Guidance Mode" to a lunar landing confirms this expectation, and calculated propellant losses are negligible. Real-time computation is possible with a small, medium speed computer (RPC 4000). This proves the feasibility of the "Iterative Guidance Mode" for on-board computation. Used for trajectory optimization, substantial savings in computer time are possible.

INTRODUCTION

The task of a guidance system generally is to direct a vehicle from a given instantaneous state to a prescribed final state. This study will describe a special system designed to handle large disturbances with a minimum energy expenditure. This is done by calculating a new optimum trajectory from any instantaneous state to the end point. By using analytical solutions of simplified problems as iteratively improving approximate solutions for the real problem, computation effort can be reduced to such an extent that real-time calculation of the optimum trajectories with on-board components becomes possible.

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Application of this scheme to a lunar landing verifies its accuracy and usefulness numerically.

Special acknowledgement is given by the author to various members of the Dynamics Analysis Branch, especially Mr. Judson J. Hart, Mrs. Doris C. Chandler, and Mr. Daniel T. Martin, for their contributions in the preparation of this paper.

COMPARISON OF VARIOUS GUIDANCE MODES

The Delta Minimum Principle

One of the classical guidance modes uses a precalculated standard trajectory as a reference. Deviations from this reference caused by external disturbances, as well as inaccurate performance, tolerances, misalignments, etc., are sensed, and the vehicle is returned to the reference.

The term "trajectory," just like the terms "state," "condition," or "point," as used before, does not only define the geometric shape, but may include any other state variables which may be of significance, e.g., velocity vector or components, time, or instantaneous mass.

The method of keeping deviations from a reference small was, under the name "Delta Minimum Principle," used for the Juno launch vehicles which carried the Explorer satellites. Its main advantage is simplicity of the computing operations, since higher order terms of the steering equation can be neglected.

Deviations from the reference can usually be kept sufficiently small if the guidance system is continuously active and if the disturbances at no point exceed the system's capability.

The Path-Adaptive Guidance Mode

If the guided phase of the trajectory is preceded by an extended unguided interval, e.g., landing on the moon (direct or through orbit) after about three days of unpowered transit, or if for other reasons large deviations have occurred, it may become very uneconomical or even impossible to return to the standard trajectory. In such a case, it is preferable to compute a new trajectory from the actual conditions to the prescribed final state. Like the original standard trajectory, this secondary reference has to be optimized, e.g., by calculus of variation. The next logical step is to abandon the concept of a reference trajectory completely and to calculate at each instance the thrust level and direction that lead in optimum fashion to the desired end condition. This approach, called the "Path-Adaptive Guidance Mode" was pointed out by Dr. R. Hoelker and his co-workers [1, 2]* for the guidance of space vehicles like the Saturn.

*Numbers in brackets indicate references at the end of the report.

The considerable computation effort spent in the optimization of even a single reference trajectory makes it impossible to do all the calculations in real time on board. Therefore, a large family of trajectories is being precalculated and evaluated before the flight, and the steering equation as a function of several variables derived and stored in the on-board guidance computer. Another possible solution for an on-board computation will be given below.

The Iterative Guidance Mode

Analytical solutions for optimum trajectory equations are possible for certain simplified cases. These analytical solutions are also approximate solutions for the general, i.e., not simplified, case. As the vehicle proceeds toward the target, the solution of the trajectory equations is repeated over and over. If the simplifications are chosen so that they converge toward the real conditions as the end point is approached, each recalculation of the trajectory will not only correct errors caused by external disturbances, but will also step-wise reduce those resulting from the original simplifications.

This iterative mode was applied to a (calculated) lunar landing problem and found to give solutions very close to a time optimum, even for considerable initial errors.

In the derivation of the equations, graphical methods were used extensively. The resulting case of understanding helped considerably toward the formulation of the problem and the proper choice of simplifications.

DERIVATION OF THE EQUATIONS

Optimization with Calculus of Variation

As mentioned at the beginning, the terminal state of the vehicle is defined not only by its position but also by the velocity vector, time of arrival, mass (respectively propellant consumption), etc. Some of these values are prescribed by the mission, e.g., height and velocity vector in the case of an ascent to orbit. All but one of the remaining variables (or degrees of freedom) are then chosen in such a way that the remaining one, usually the final mass or the final velocity, becomes an extremal.

One of the two control variables available to shape the trajectory, the thrust level, may for the time being be considered as constant, i.e., determined by the power of a selected engine.

Making some additional simplifying assumptions, a homogeneous gravitational field, vacuum conditions, and constant specific impulse, Fried [3] and Lawdon [4] show an analytical solution for the remaining independent variable, the thrust direction.

$$\tan\varphi = \frac{\partial R/\partial \dot{y}_T + (T - t)\ \partial R/\partial y_T}{\partial R/\partial \dot{x}_T + (T - t)\ \partial R/\partial x_T} = \frac{a + bt}{c + dt}\ . \tag{1}$$

In an important special case, the injection into a circular orbit, the range over ground to the injection point x_T is not necessarily specified. To have maximum mass, the partial derivative becomes

$$\partial R/\partial x_T = 0\ . \tag{2}$$

Subsequently, the steering equation takes the form

$$\tan\varphi = \frac{\partial R/\partial \dot{y}_T + (T - t)\ \partial R/\partial y_T}{\partial R/\partial \dot{x}_T} = a' + b't. \tag{3}$$

The same equation is valid if any straight line is prescribed as locus for the terminal point. The x, y coordinate system has then to be replaced by a rotated ξ, η system, the ξ axis of which is parallel to the straight line, and the thrust angle φ is measured against the ξ axis.

The end conditions of a trajectory, following steering equation (3), were calculated in analytical form by Ehlers [6]. No such solution for the inverted problem, the computation of the coefficients a' and b' that lead to a given end condition, seems to be available. Approximate solutions, e.g., by serial development and truncations, are possible. However, geometric relations can be exploited to derive another solution, which will be pursued in this report.

Graphical Solution if End Position is not Specified

Some understanding of the ideas that lead to the formulation of the iterative guidance mode will contribute to its general usefulness and its adaptation to various missions. Therefore, the derivation of equations, and particularly the use of graphical methods, is presented in considerable detail for some typical applications.

If the location of the end point is of no concern, Figure 1 allows a quick derivation of the equations. The inertial velocity, v_i is shown for clarity as the chain of arrow connecting point P_3 to P_4', P_4'', or P_4''', respectively, where

$$\vec{v}_i = \sum \vec{F}/m\ \Delta t\ , \tag{4}$$

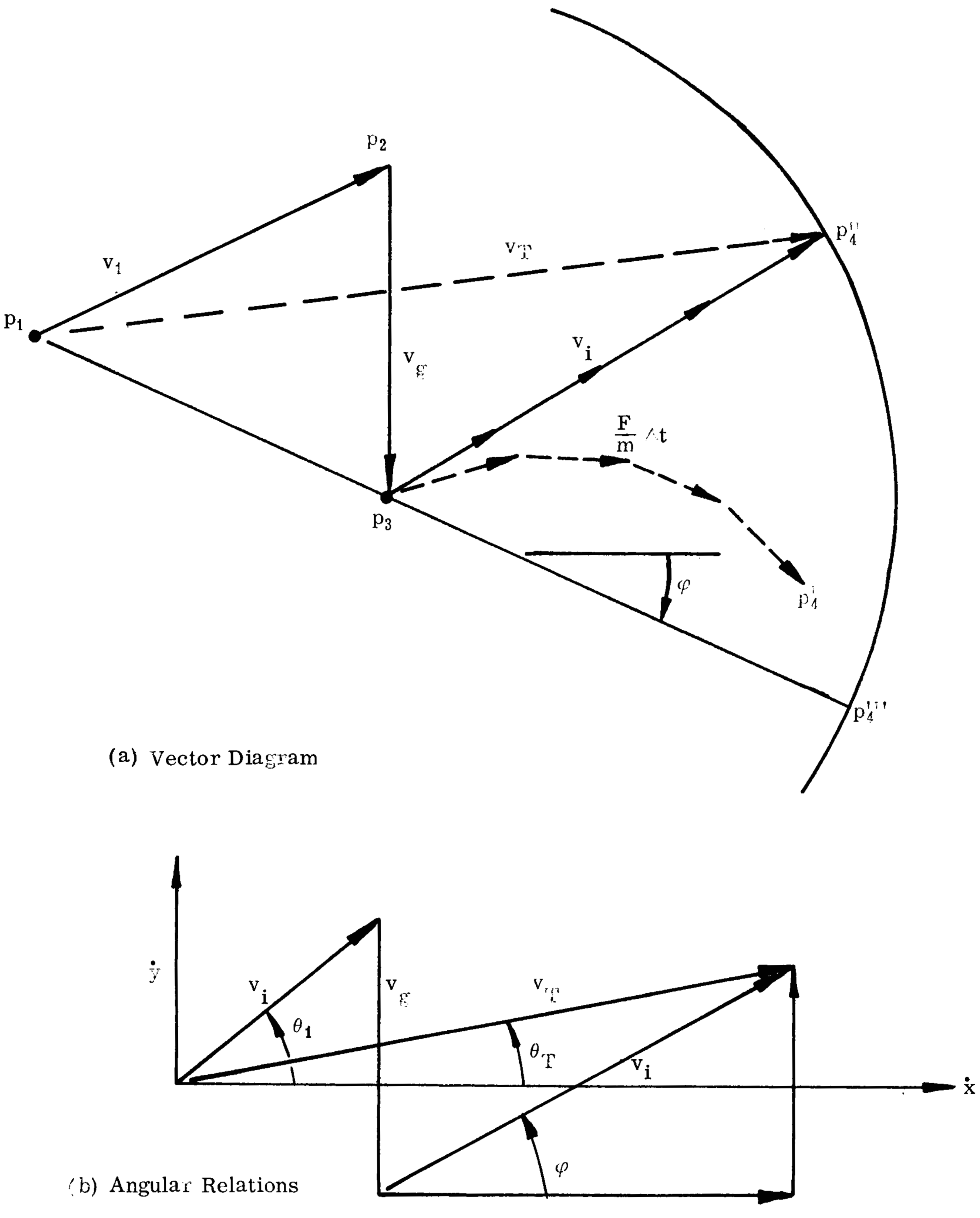

FIGURE 1. OPTIMIZATION FOR FREE CHOICE OF END LOCATION

instead of the true integral form. It is obvious that the distance $P_3P_4''' = |v_i|$ is largest if the "chain" is straight. This means, however, that the thrust angle φ is constant. With $v_1 = P_1P_2$ representing the initial velocity and $v_g = P_2P_3$ representing the gravitational contribution, the locus for the total velocity vector v_T is a circle with P_3 as center and v_i as radius. Prescribing the direction of v_T provides the necessary second condition for the end point P_4'' of v_T and v_i. If the velocity direction is free, the requirement for maximum velocity will place the end point to P_4'''.

From the well known rocket equation, we determine

$$v_i = \int \frac{F}{m}\, dt = c^* \int \frac{\dot{m}}{m_1 + \dot{m}t}\, dt \tag{5a}$$

$$= c^* \ln \frac{m_1}{m_1 + \dot{m}t} \tag{5b}$$

$$= c^* \ln \frac{\tau}{\tau - t} \tag{5c}$$

where the auxiliary variable

$$\tau = -\frac{m_1}{\dot{m}} \tag{5d}$$

gives the time at which the vehicle would have completely burned.

From Figure 1b, Equation (5c) and

$$v_g = \dot{y}_g = gT, \tag{6}$$

it follows that

$$\dot{x}_i = c^* \ln \frac{\tau}{\tau - T} \cos \varphi = v_T \cos \theta_T - \dot{x}_1 \tag{7}$$

$$\dot{y}_i = c^* \ln \frac{\tau}{\tau - T} \sin \varphi = v_T \sin \theta_T - \dot{y}_i + gT. \tag{8}$$

These two equations can be solved simultaneously for either T and φ or for v_T and φ, as the case may require.

Approximate Solution if One Coordinate of the End Point is Specified

The additional constraint in the end condition requires an additional degree of freedom added to the steering equation, which can be provided by using the form of Equation (3), somewhat rewritten:

$$\varphi = \tilde{\varphi} - (K_1 - K_2 t) , \tag{9}$$

where $\tilde{\varphi}$ is the constant thrust angle that satisfies the velocity conditions. Equations (7) and (8), and K_1 and K_2 will be chosen so that the position constraint y_T is satisfied without disturbing the end velocity condition $\dot{y}_T$. For small values of $(K_1 - K_2 t)$, the equations of motion can be approximated.

$$\ddot{y}_i = F/m \sin [\tilde{\varphi} - (K_1 - K_2 t)] = \frac{c^*}{\tau - t} \sin [\tilde{\varphi} - (K_1 - K_2 t)] \tag{10a}$$

$$\ddot{y}_i = \ddot{\tilde{y}}_i - \frac{c^*}{\tau - t} (K_1 - K_2 t) \cos \tilde{\varphi} \tag{10b}$$

$$\dot{y}_i = \dot{\tilde{y}}_i - c^* \cos \tilde{\varphi} \left[+K_1 \ln \frac{\tau}{\tau - t} - K_2 \left(\tau \ln \frac{\tau}{\tau - t} - t\right)\right] \tag{11a}$$

$$\dot{y}_i(T) - \dot{\tilde{y}}_i(T) = c^* \cos \tilde{\varphi} \left[- K_1 \ln \frac{\tau}{\tau - T} + K_2 \left(\tau \ln \frac{\tau}{\tau - T} - T\right)\right] = 0 \tag{11b}$$

$$y_i = \tilde{y}_i - c^* \cos \tilde{\varphi} \left[K_1 [t - (\tau - t) \ln \frac{\tau}{\tau - t}] + K_2 \left\{ \frac{t^2}{2} - \tau [t - (\tau - t) \ln \frac{\tau}{\tau - t}] \right\} \right] \tag{12a}$$

$$y_T(T) = y_1 + \dot{y}_1 T - \frac{1}{2} g T^2 + y_i(T) = y_T (T)_{Nom.} \tag{12b}$$

Equations (11) and (12) can be solved simultaneously. If necessary, the horizontal velocity component can be corrected by recomputing the flight time.

Both End Point Coordinates Specified

As Equation (1) shows, an additional degree of freedom, in the form of the constant c, is available to allow one more constraint. (The constants c and d are not free since by division either can be made unity.) In practice, however, any shifting of the end point in the "average" flight direction by changing the thrust angle is rather ineffective; this should rather be accomplished by adjusting the thrust level either by continuous throttling or, stepwise, by turning all or part of the engines on and off.

For this study, it will be assumed that the constant thrust level which would satisfy the end conditions will be recomputed at adequate intervals and the engine adjusted accordingly.

A LUNAR LANDING SCHEME

Mission and Ground Rules

Only a rather general idea, subject to rather restrictive simplifications, has been presented so far. The easiest way to check its practical usefulness is to apply it to a specific mission, e.g., the automatic landing of a space vehicle on the moon. We guide the vehicle from a lunar orbit, either a nominally circular one or a Hohmann transfer ellipse, to a predetermined point on the surface, and then compare the trajectories for nominal and strongly disturbed initial conditions. The system can then be judged by the accuracy with which the landing point and zero velocity are reached, the amount of fuel used above that for a true optimum path, and the magnitude of the maneuvers required, especially engine throttling.

Since the information available to determine the instantaneous state probably will be limited, we will restrict ourselves to the use of D, the line of sight distance to the landing site; $\dot{D}$, its rate of change; ϵ, angle between the line of sight direction to landing site and the local horizontal; $\dot{\epsilon}$, its rate of change; and a_i, the inertial acceleration.

While the amount of information is considered very carefully, the means of obtaining it (radio, optical, inertial, etc.) are beyond the scope of this study. Instrumentation errors are not considered either, but the trend of errors to become smaller as the measured value decreases will hopefully keep them within reason.

The Guidance Equation

The flight geometry shown in Figure 2 shows the primary xy coordinate system, pointing in the local horizontal and vertical at the landing site. The ξ-η system is formed by rotating the x-y system through the variable angle ϵ with the negative ξ-axis pointing toward the vehicle.

The instantaneous coordinates of the vehicle are x_1 (shown to be negative), y_1, ξ_1 (negative), and $\eta_1 = 0$. Since the landing site is at the point of origin, all nominal values at the terminal point are zero. As the gravitation changes in direction and magnitude, mean values were used for both:

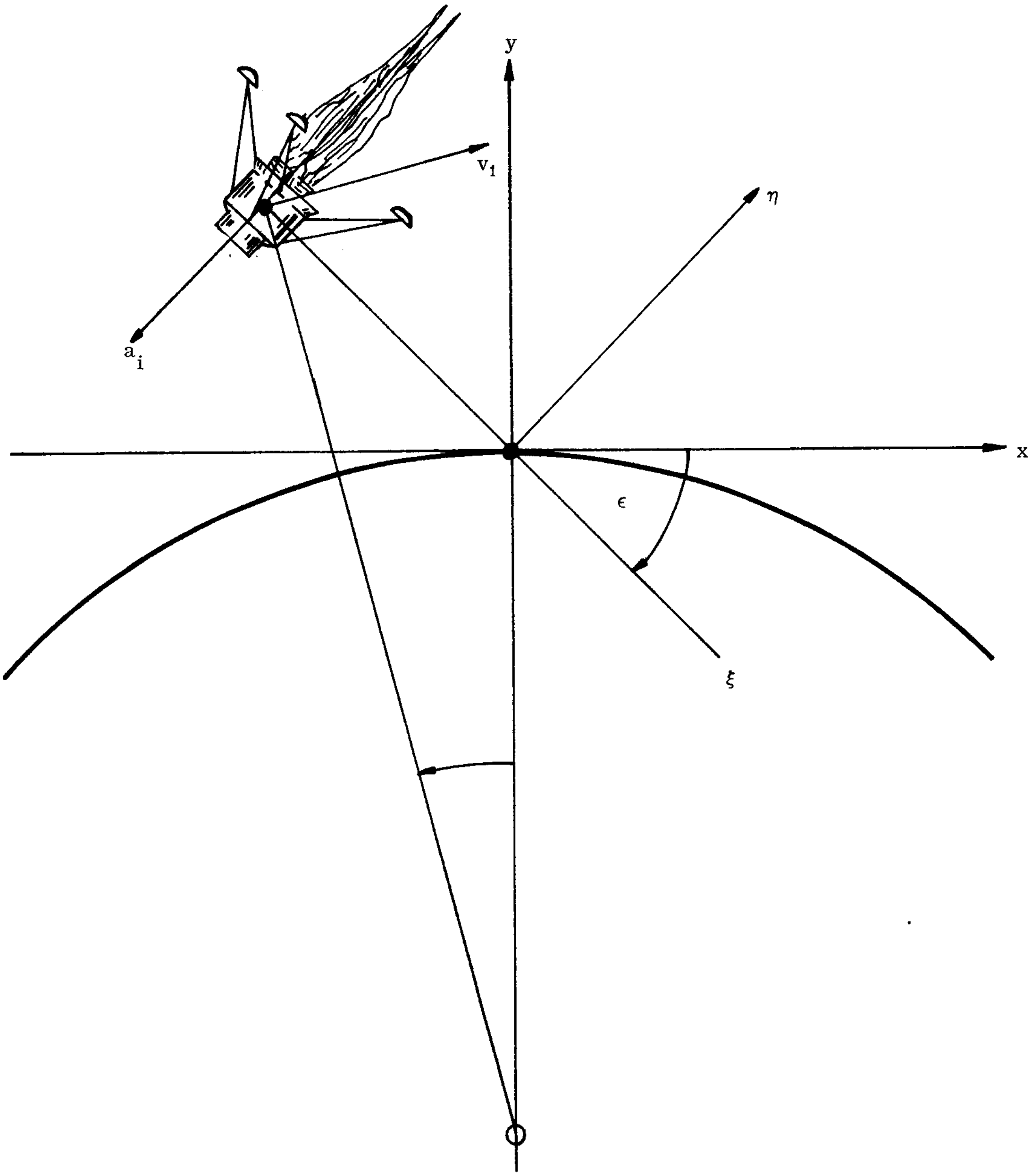

FIGURE 2. LUNAR FLIGHT GEOMETRY

$$\phi^* = \frac{1}{2}\,\phi_1 = \text{arc tan}\,\frac{D_1 \cos\,\epsilon_1}{r_L + D \sin\,\epsilon_1} \tag{13}$$

$$g^* = \frac{1}{2}\,(g_{L_1} + g_L) \approx g_L \left[1 - \frac{y_1}{r_L} - \frac{1}{2}\left(\frac{D \cos\,\epsilon}{r_L}\right)^2\right]. \tag{14}$$

An attempt was first made to calculate the thrust level from the condition that $\dot{\xi}$ and ξ must simultaneously become zero, but it was found more simple and accurate to use the precalculated value F^* for a nominal trajectory as initial value, calculate the ξ^* that would result from its use, and correct the thrust level to

$$F_1 = F^* \frac{\xi^*}{-\xi_1} . \tag{15}$$

For each integration step, the thrust from the previous step is used as first approximation. The other steps shown in the flow diagram (Figure 3) are self-explanatory. A detailed derivation of the equations is given in a NASA report [5].

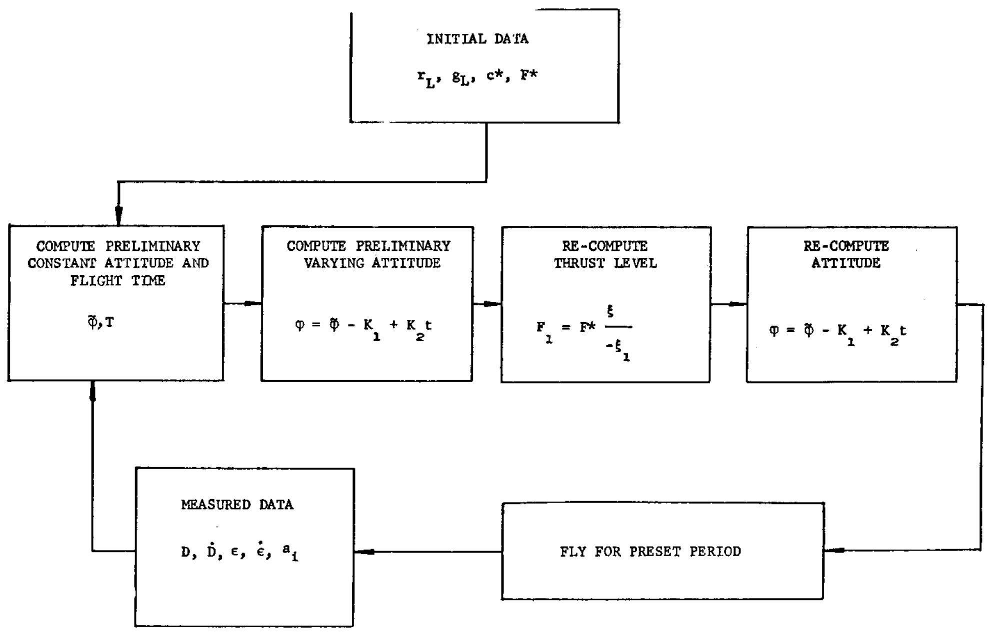

FIGURE 3. FLOW DIAGRAM

The interval for recomputing the guidance equation was arbitrarily set at 10 seconds. There are indications that a considerably longer step would have been permissible.

First Check Case: Direct Landing from Circular Orbit

A circular parking orbit with a nominal height of 100 km is selected as initial condition for the landing. The vehicle has an initial mass of 30,000 kg, a nominal thrust of 9000 kp, specific impulse of 420 sec, and in the nominal case a landed mass of 18,572 kg.

Table I shows the effect of probable error sources on the accuracy of the guidance. The perturbations consist of distortions of the parking orbit into an ellipse, variations in the orientation of the ellipse, and errors in ignition time.

TABLE 1. ERROR SUMMARY FOR DIRECT LANDING FROM PARKING ORBIT

Case	Peri-selenum km	Apo-selenum km	Angle between Periselenum & Land-Site deg	Ignition Time Error sec	Thrust Variation max. kp	X m	Landing Errors y m	$\dot{y}$ m/s
I	100	100	0	0	±320	-.12	-.09	-.02
II	100	150	180	0	±850	-.07	-.05	-.01
III	50	100	0	0	± 60	-.10	-.12	-.02
IV	100	150	0	0	±310	-.07	-.05	-.01
V	50	100	180	0	±340	-.09	-.06	-.01
VI	75	125	90	0	±320	-.12	-.07	-.02
VII	75	125	270	0	±320	-.09	-.07	-.02
VIII	100	100	0	+10	±310	-.10	-.07	-.02
IX	100	100	0	-10	±320	-.12	-.01	-.02

Table 1 shows that in all cases the errors at landing are negligible. The payload loss, compared to an optimum descent from the initial conditions, was within the computing accuracy of about 10 kg, and therefore is not shown. The thrust level variations shown are the maximum deviations occurring at any time. They are always within ±10 percent of the nominal value. Three typical trajectories are shown in Tables 2, 3, and 4.

TABLE 2. DESCENT FROM NOMINAL PARKING ORBIT (CASE I)

t sec	X km	y km	φ deg	v m/s	F kp	h km
60	-448.9	44.3	227.9	1695.3	9005	100.0
100	-387.0	57.5	217.7	1627.3	8912	98
140	-328.5	65.4	208.1	1548.4	8844	95.1
180	-273.8	68.7	199.0	1459.5	8796	89.4
220	-223.5	68.1	190.4	1361.5	8762	81.8
260	-178.0	64.1	182.2	1255.1	8737	72.9
300	-137.5	57.6	174.2	1140.6	8720	62.8
340	-102.3	49.2	166.6	1018.4	8708	52.1
380	-72.5	39.6	159.2	888.6	8701	41.1
420	-48.1	29.7	151.9	751.3	8696	30.4
460	-29.0	20.1	144.8	606.3	8694	20.4
500	-14.9	11.6	137.8	453.5	8693	11.7
540	-5.7	5.0	130.9	292.9	8693	5.0
580	-0.9	0.9	124.0	124.3	8693	0.9
608.3	0.0	0.0	119.3	0.0	8693	0.0

TABLE 3. DESCENT FROM DISTURBED PARKING ORBIT (CASE II)

t sec	X km	y km	φ deg	v m/s	F kp	h km
108.8	-481.0	86.8	246.3	1656.8	9037	149.2
148.8	-420.0	100.2	232.2	1630.0	8773	147.6
188.8	-361.4	107.7	219.6	1585.3	8586	142.7
228.8	-306.0	109.9	208.2	1525.1	8467	135.1
268.8	-254.5	107.6	197.8	1451.9	8386	125.1
308.8	-207.4	101.6	188.1	1367.5	8331	113.3
348.8	-165.0	92.6	179.0	1272.9	8291	100.0
388.8	-127.5	81.3	170.4	1169.0	8264	85.8
428.8	-95.2	68.6	162.1	1056.2	8244	71.1
468.8	-67.9	55.1	154.2	935.0	8231	56.4
508.8	-45.7	41.7	146.5	805.4	8222	42.3
548.8	-28.3	29.1	139.0	667.4	8216	29.3
588.8	-15.5	17.9	131.7	521.0	8214	18.0
628.8	-6.8	8.9	124.6	366.1	8213	8.9
668.8	-1.8	2.7	117.5	202.9	8213	2.7
715.9	0	0	109.4	0	8213	0

TABLE 4. DESCENT FROM DISTURBED PARKING ORBIT (CASE III)

t sec	X km	y km	φ deg	v m/s	F kp	h km
113.6	-432.7	-2.4	207.3	1707.8	8999	50.7
153.6	-369.9	11.3	209.5	1607.4	8983	50.0
193.6	-311.1	20.8	194.7	1501.3	8970	48.1
233.6	-256.7	26.7	188.6	1389.6	8961	45.3
273.6	-206.9	29.4	182.7	1272.4	8954	41.4
313.6	-162.2	29.3	176.8	1149.6	8949	36.7
353.6	-122.7	27.1	171.1	1021.3	8946	31.3
393.6	-88.6	23.2	165.4	887.3	8944	25.4
433.6	-60.0	18.3	159.7	747.6	8942	19.3
473.6	-37.0	13.0	154.1	601.8	8942	13.4
513.6	-19.6	7.8	148.6	449.8	8942	7.9
553.6	-7.8	3.5	143.0	291.3	8942	3.5
593.6	-1.4	.7	137.5	125.9	8943	.7
622.9	0	0	133.5	0	8943	0

Second Check Case: Landing from a Hohmann Transfer Ellipse

A nominal periselenum of 20 km as standard ignition point is chosen. It is varied similarly as in First Check Case, except that the periselenum altitude is changed ±100 percent (Table 5). The vehicle is the same as in Case I except that the thrust has a nominal value of 15,000 kp and the landed mass is 19,522 kg.

TABLE 5. ERROR SUMMARY FOR LANDING FROM HOHMANN ELLIPSE

Case	Peri-selenum km	Apo-selenum km	Angle fr. Periselenum to Land-Site deg	Time Error sec	Thrust Variation Max. kp	Landing Errors X m	y m	y m/s
I	20	100	-8.5	0	±30	-.14	-.28	-.04
II	40	100	-8.5	0	±170	-.29	-.38	-.07
III	0	100	-8.5	0	±10	-.01	-.05	-.01
IV	20	100	-18.5	0	±40	-.15	-.31	-.05
V	20	100	+1.5	0	±30	-.14	-.28	-.04
VI	20	100	-8.5	+10	+1100 -30	-.17	-.34	-.05
VII	20	100	-8.5	-10	+30 -950	-.10	-.21	-.03

Case III demonstrates rather dramatically the power of the system. The vehicle grazes the surface when power is applied. Still, it recovers and lands with perfect accuracy (Figure 4 and Table 8). In addition, Cases III and IV are equivalent to missing the time for the Hohmann kick by about ±3 minutes. Tables 6, 7, and 8 show some more trajectory details.

TABLE 6. DESCENT FROM NOMINAL HOHMANN ELLIPSE (CASE I)

t sec	X km	y km	φ deg	v m/s	F kp	h km
54.3	-259.1	0.8	199.0	1702.9	15000	20.0
94.3	-196.0	8.3	192.1	1515.2	14987	19.3
134.3	-140.6	11.7	185.2	1314.4	14978	17.3
174.3	-93.7	11.6	181.5	1099.8	14973	14.1
214.3	-55.8	9.3	171.9	871.1	14970	10.2
254.3	-27.4	5.8	165.3	627.0	14968	6.0
294.3	-8.8	2.3	158.8	366.7	14968	2.3
314.3	-3.4	0.9	155.6	230.0	14968	0.9
346.5	-0.0	0.0	150.4	0.0	14968	0.0

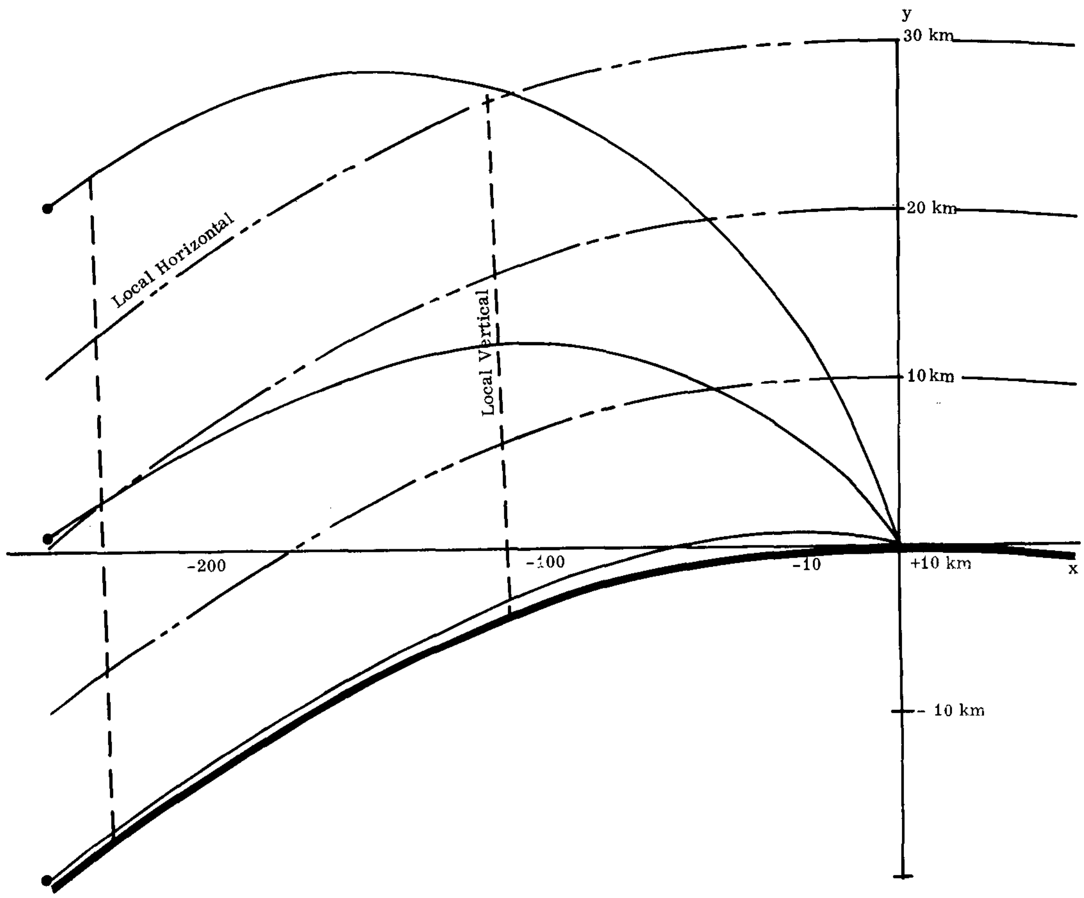

FIGURE 4. TYPICAL TRAJECTORIES

TABLE 7. DESCENT FROM DISTURBED HOHMANN ELLIPSE (CASE II)

t sec	X km	y km	φ deg	v m/s	F kp	h km
54.2	-263.9	20.3	213.3	1698.0	15011	40.0
94.2	-201.0	27.0	202.4	1534.5	14947	38.4
134.2	-145.5	28.2	192.0	1351.5	14905	34.1
174.2	-98.1	25.0	181.2	1149.8	14879	27.7
214.2	-59.6	19.0	172.1	929.6	14865	20.0
254.2	-30.5	11.8	162.6	691.0	14859	12.1
294.2	-11.1	5.1	153.2	433.8	14857	5.1
334.2	-1.3	0.7	143.9	157.5	14857	0.7
355.8	-0.0	0.0	139.1	0.0	14857	0.0

TABLE 8. DESCENT FROM DISTURBED HOHMANN ELLIPSE, GRAZING SURFACE (CASE III)

t sec	X km	y km	φ deg	v m/s	F kp	h km
51.4	-259.1	-19.4	184.2	1696.5	15003	0.0
91.4	-195.6	-10.8	181.3	1498.0	15005	0.2
131.4	-140.1	-4.9	178.3	1289.8	12007	0.7
171.4	-93.0	-1.4	175.4	1070.6	15008	1.1
211.4	-54.9	0.3	172.5	839.3	15009	1.2
251.4	-26.4	0.7	169.6	594.3	15009	0.9
291.4	-8.0	0.4	166.6	333.9	15010	0.4
339.2	0.0	0.0	163.2	0	15010	0.0

CONCLUSIONS

No attempt is made to prove the validity of the iterative guidance scheme in a general way. However, when used for a specific application, a lunar landing, all requirements previously stated are satisfied. It handles disturbances beyond those expected in a real flight with a minimum of measurements. Payload loss is negligible, the landing accuracy does not deteriorate, and control requirements, especially thrust control, are acceptable.

At the same time, the computation effort is modest. Calculation time for the steering equations (including trajectory calculation) for one integration step is 10 seconds on a small, medium speed computer, the RCP 4000, i.e., in real time. Storage requirements of 1300 words are well within the capacity of these computers.

The important question of how to measure the required variables is beyond the topic of this report. Other questions that remain are (1) the extension to three dimensions, which should not pose any major difficulties, (2) some improvement of the computer program, and (3) an analysis of the effects of instrument errors.

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