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Space Guidance Analysis Memo #57

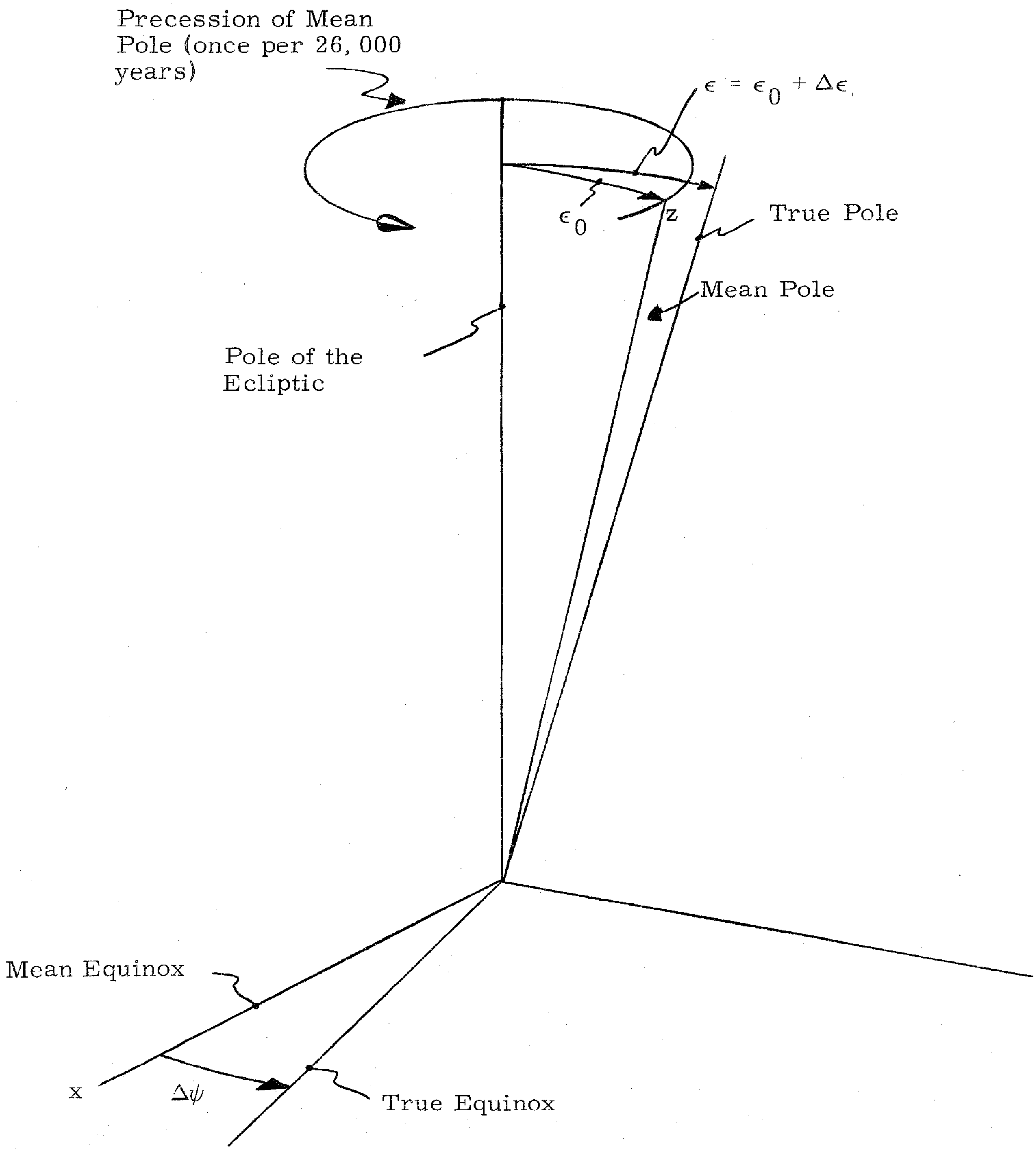
TO: SGA Distribution
FROM: Larry D. Brock
DATE: September 16, 1963
SUBJECT: Coordinate System Used in the Moon Position Subroutine

The coordinate system that is used in the moon position subroutine is defined by the mean equator and the mean orbit of the earth (the ecliptic) at the nearest beginning of a Besselian year. For example, the coordinate system used for a mission that begins in the interval from July 1, 1963 through June 30, 1964 would use the mean equator and equinox at the beginning of the Besselian year 1964 (denoted by 1964. 0) which occures January 1. 314. The x axis is defined along the ascending node of the ecliptic on the equator (the equinox), the z axis is defined along the mean pole, and the y axis is defined so as to complete a right handed set.

The coordinate system is defined by the mean equator and ecliptic as of a certain date because the true equator and ecliptic are in constant motion.[1] The motion of the equator is caused primarily by the gravitational attraction of the sun and moon of the earth's equatorial buldge and is divided into two components. One component, called luni-solar precession, defines the position of the mean pole and consists of a smooth long period rotation of the mean pole around the pole of the ecliptic with a period of about 26, 000 years.

The other component, called nutation, describes the true pole relative to the mean pole and consists of many oscillatory terms with periods ranging from 19 years to a few days. The primary term is a irregular circular motion of the true pole about the mean pole with a period of about 19 years and an amplitude of about 9''. Nutation is described by two quantities; nutation in longitude ($\Delta\psi$) and nutation in obliquity ($\Delta \epsilon$). These quantities are shown in Fig. 1 and are defined by a series similar to the Brown series for the position

[1] Explanatory Supplement to the Astronomical Ephemeris and American Ephemeris and Nautical Almanac, Her Majesty's Stationery Office, 1961, pp. 28.

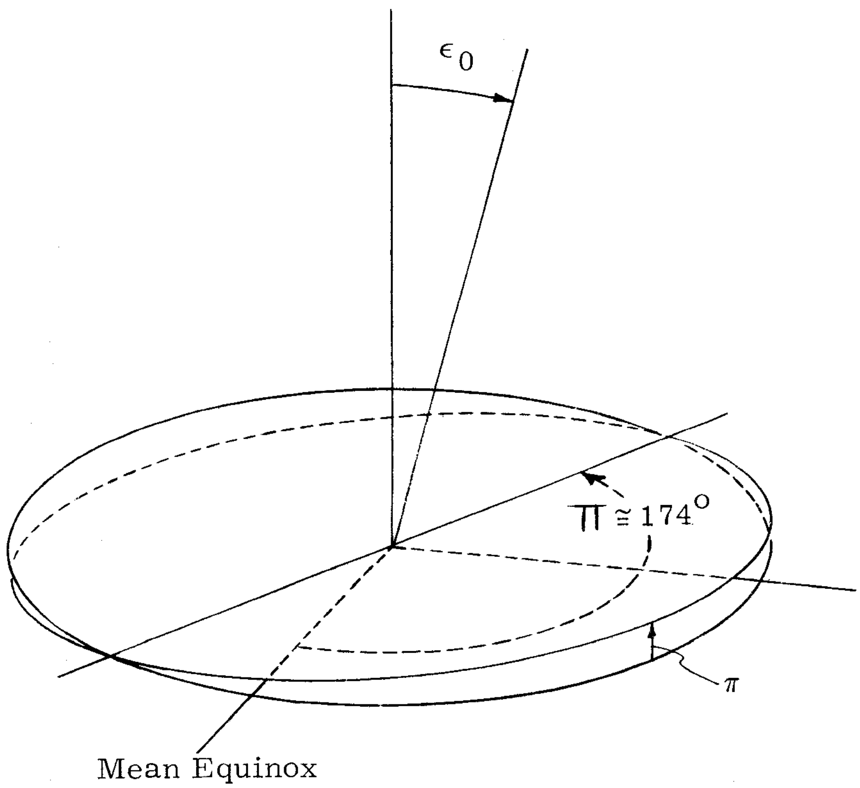


The Motion of the Earth's Pole

Figure 1

of the moon. The Nautical Almanac Office uses all terms of $0\overset{''}{.}0002$ or greater. These series are given on pages 44 and 45 of the Explanatory Supplement to the Ephemeris. There are 69 terms for $\Delta\psi$ and 40 terms for $\Delta\epsilon$.

The motion of the ecliptic is called planetary precession and is caused by the gravitational force of the planets on the earth as a whole. It consists primarily of a slow rotation π ($0.47''$ per year) of the ecliptic about a diameter that is also slowly rotating (Π). This precession is shown in Fig. 2.



The Motion of the Ecliptic
Figure 2

The precession of the ecliptic is combined with the precessional component of the motion of the equator to form the total precession which defines the mean equator and ecliptic.[2] A vector in rectangular coordinates referred to the equinox of one epoch (1950.0 + T_0) can be transformed to the rectangular coordinates of the vector referred to another epoch (1950.0 + T_0 + T) by the equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_x & Y_x & Z_x \\ X_y & Y_y & Z_y \\ X_z & Y_z & Z_z \end{bmatrix} \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} \qquad (1)$$

where

$$X_x = 1 - (.00029696 + .00000026\, T_0)\, T^2 - .00000013\, T^3$$

$$Y_x = -X_y = -(.02234941 + .00001355\, T_0)\, T - .00000676\, T^2 + .00000221\, T^3$$

$$Z_x = -X_z = -(.00971690 + .00000414\, T_0)\, T + .00000207\, T^2 + .00000096\, T^3$$

$$Y_y = 1 - (.00024975 + .00000030\, T_0)\, T^2 - .00000015\, T^3$$

$$Y_z = Z_y = -(.00010858 + .00000002\, T_0)\, T^2$$

$$Z_z = 1 - (.00004721 - .0000000\, T_0)\, T^2$$

where T_0 and T are in tropical centuries.

Some of the advantages of using a coordinate system referred to the mean equinox at the nearest beginning of the year are the following:

1. The precession of the earth can usually be neglected in computing the earth oblateness acceleration. Since the reference equinox is always within six months of the mean equinox of date, the maximum error in the pole due to precession is about 10''.

[2] Ibid pp. 34

2. Several of the terms in equation (1) can be neglected in transforming from earth fixed coordinates to the reference coordinates.
3. This is the coordinate system in which much of the data from the Nautical Almanac Office is given such as the position of the moon, sun, stars, etc. Thus possible errors are avoided in transforming these data.