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Space Guidance Analysis Memo #52

TO: SGA Distribution
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SUBJECT: Midcourse Guidance

1. Introduction

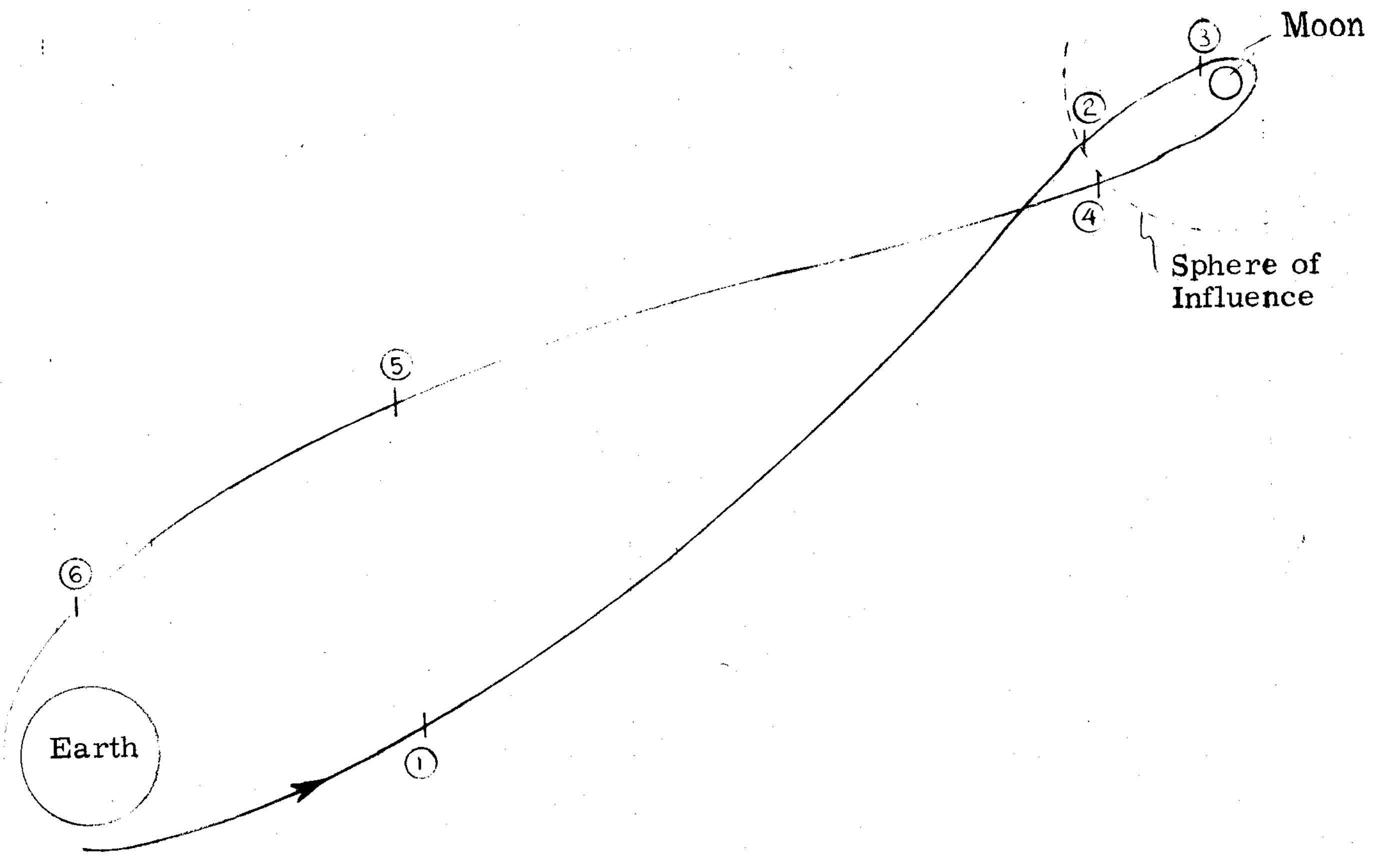
This memo describes the procedures that have been developed for determining velocity corrections during the midcourse phase of the lunar mission. A navigation system similar to that described in "A Statistical Optimizing Navigation Procedure for Space Flight" by Dr. Richard H. Battin is assumed and will not be discussed.

The basic philosophy for the calculation of a velocity correction is as follows:

1) Integrate the present position and velocity estimates to obtain the end conditions that would result if no correction were made.
2) Determine the desired terminal conditions.
3) Calculate the conic velocities connecting the present position to both the obtained and the desired final conditions.
4) The velocity correction is then the difference between these two conic velocities.

All velocity correction procedures discussed below will use this philosophy or extensions thereof.

We consider the problem of guiding a spacecraft in a circumlunar trajectory as illustrated in Figure 1. Previous studies have shown that the optimum velocity correction schedule is approximately as shown in Figure 1 with corrections being made at the numbered points. It should be noted that points 2 and 4 are the entry and the exit points respectively of the sphere of influence of the moon.



2. Types of Velocity Corrections

A velocity correction procedure is specified by the desired terminal conditions. The three different types of corrections which will be considered are as follows:

1) Position Guidance: The desired terminal conditions are a position vector at a time. This type of velocity correction is not recommended for use near a planet because of the resulting large velocity corrections and large velocity deviations upon arrival at the target.

2) Perilune Guidance: The desired terminal conditions are a perilune distance (not a vector) and a fixed inertial plane. The time of arrival at perilune is not constrained. Theoretically, the desired plane should not be fixed but should rotate with the moon. However, the change in time of arrival at perilune combined with the moon's

rotation leads to deviations that are smaller than the uncertainties. Hence, it is sufficient to aim for a fixed plane when approaching perilune.

3) Perigee Guidance: The desired terminal conditions are a perigee distance and a landing site fixed to the earth. This type of velocity correction is an extension of perilune guidance with the plane determined so that the spacecraft will rendezvous with the landing site.

These three types of corrections are discussed in detail in the remainder of this memo.

3. Position Guidance

To calculate the velocity correction apply the following steps:

1) Integrate the present position and velocity estimates to the given time obtaining the final position.
2) The desired final position is given.
3) Calculate the conic velocities by solving Lambert's problem iteratively.
4) Difference the conic velocities to obtain the velocity correction.

4. Perilune Guidance

Let $\underline{r}_o$ and $\underline{v}_o$ be the present position and velocity estimates, r_p the desired perilune distance, and $\underline{\eta}$ the unit normal to the desired plane. The calculations for perilune guidance are then as follows:

1) Integrate $\underline{r}_o$ and $\underline{v}_o$ to $\underline{r}$ and $\underline{v}$ sufficiently near perilune, and calculate the obtained perilune and central angle.

$$p = \frac{(\underline{r} \times \underline{v}) \cdot (\underline{r} \times \underline{v})}{\mu}$$

$$\frac{1}{a} = \frac{2}{r} - \frac{v^2}{\mu}$$

$$e = \sqrt{1 - \frac{p}{a}}$$

$$\cos \phi = \frac{p - r}{e\, r}$$

$$r'_p = a(1 - e)$$

$$\underline{r}'_p = \frac{e + \cos \phi}{1 + e} \underline{r} + \frac{r'_p\, r \sin \phi}{\sqrt{\mu p}} \underline{v}$$

$$\cos \theta = \underline{u}_{\underline{r}_o} \cdot \underline{u}_{\underline{r}'_p}$$

(The notation $\underline{u}_{\underline{SUB}}$ denotes a unit vector in the direction of the subscript).

2) Rotate the obtained perilune vector into the desired plane to the desired altitude maintaining the same central angle θ.

$$\beta = - \frac{\cos \theta}{1 - (\underline{\eta} \cdot \underline{u}_{\underline{r}_o})^2}$$

$$\underline{r}_p = r_p \left[\sqrt{1 + \beta \cos \theta}\, \underline{u}_{\underline{\eta}} \times \underline{u}_{\underline{r}_o} + \beta\, \underline{\eta} \times (\underline{\eta} \times \underline{u}_{\underline{r}_o}) \right]$$

3) Calculate the conic velocities connecting $\underline{r}_o$ with $\underline{r}_p$ and $\underline{r}'_p$ such that $\underline{r}_p$ and $\underline{r}'_p$ are perigees.

$$p = \frac{r_o r_p (1 - \cos \theta)}{r_p - r_o \cos \theta}$$

$$\underline{v}_c = \frac{\sqrt{\mu p}}{\sin\theta}\left[\frac{\underline{u}_{r_p}}{r_o} - \left(\frac{1}{r_p} - \frac{1 - \cos\theta}{p}\right)\underline{u}_{r_o}\right]$$

similarly for $\underline{v}'_c$

4) a. At this point the central angle θ is permitted to change so as to minimize the necessary velocity correction. Calculate the direction of insensitivity, i.e., the direction in which a velocity correction does not change pericenter (the insensitive direction is independent of the type of conic).

$$\underline{d} = -(1 - \cos\theta)^2\,\underline{u}_{r_p} + \sin\theta\,\left(1 - \cos\theta + \frac{p}{r_p}\right)\underline{u}_{r_p} \times \underline{\eta}$$

4) b. The velocity correction is then the component of $(\underline{v}_c - \underline{v}'_c)$ in the plane perpendicular to $\underline{d}$.

$$\Delta\underline{v} = (\underline{v}_c - \underline{v}'_c) - \left[(\underline{v}_c - \underline{v}'_c)\cdot\underline{u}_d\right]\underline{u}_d$$

5. Perigee Guidance

Let

$\underline{r}_o$ = present position estimate

$\underline{v}_o$ = present velocity estimate

t_o = present time

r_p = desired perigee

t_{p_o} = nominal arrival time at perigee

$\underline{r}_{L_o}$ = position of landing site at the time the nominal trajectory arrives there.

α_o = nominal angle from perigee to $\underline{r}_{L_o}$.

It is assumed that during the re-entry phase the spacecraft travels at circular orbital speed, i.e., sixteen times as fast as a point fixed to the earth.

1) a. Integrate t_o, $\underline{r}_o$, and $\underline{v}_o$ to t, $\underline{r}$, and $\underline{v}$, as in Step 1 of perilune guidance to obtain r'_p, $\underline{r}'_p$, and cos θ.

b. Estimate the time of arrival at perigee.

$$E = \cos^{-1}\left(\frac{1 - \frac{r}{a}}{e}\right)$$

$$T_p = T + \sqrt{\frac{a^3}{\mu}}\,(E - e \sin E)$$

2) a. Determine the desired plane by solving iteratively the following set of equations for $\underline{r}_L$.

$$A = \frac{\pi}{12}\,(T_p - T_{p_o}) + \frac{\alpha - \alpha_o}{16}$$

$$\underline{r}_L = \begin{pmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix} \underline{r}_{L_o}$$

$$\alpha = \begin{cases} \cos^{-1}(\underline{u}_{\underline{r}_o} \bullet \underline{u}_{\underline{r}_L}) - \theta & \text{if angle } (\underline{r}_o, \underline{r}_L) < 180^\circ \\ 2\pi - \cos^{-1}(\underline{u}_{\underline{r}_o} \bullet \underline{u}_{\underline{r}_L}) - \theta & \text{if angle } (\underline{r}_o, \underline{r}_L) > 180^\circ \end{cases}$$

Then

$$\eta = \begin{cases} +\,\underline{u}_{\underline{r}_o} \times \underline{r}_L & \text{if angle } (\underline{r}_o, \underline{r}_L) < 180^\circ \\ -\,\underline{u}_{\underline{r}_o} \times \underline{r}_L & \text{if angle } (\underline{r}_o, \underline{r}_L) > 180^\circ \end{cases}$$

b. Determine $\underline{r}_p$.

Step 2 of perilune guidance.

3. Calculate the conic velocities.

Step 3 of perilune guidance.

4. Difference the conic velocities.

Step 4 of perilune guidance.

6. Circumlunar Guidance Procedure

Returning to Figure 1, consider the following procedure for guidance in a circumlunar flight. At point 1, position guidance to point 2; at points 2 and 3, perilune guidance; at points 4, 5, and 6, perigee guidance.

Computer simulation of this guidance scheme produced excellent results for the translunar half of the flight; however, for the transearth half, unreasonably large velocity corrections were required at all three points. A large correction might be expected at point 4, but not at points 5 and 6 since the same criteria are to be satisfied there as at point 4.

7. Improved Perigee Guidance

It was then observed that a more accurate estimate of the time of arrival at perigee (Step 1b of perigee guidance) was required because the velocity correction itself alters the arrival time. The change in perigee time due to the correction can be approximated very accurately by the following empirically determined formula

$$\Delta t_p = 16 \times 10^{-10} r_o (r_p - r'_p)$$

Incorporation of this estimate into Step 1b of perigee guidance produced but a moderate reduction in the size of corrections 5 and 6. Under perfect translunar injection with perfect measurements, the six velocity corrections in miles per hour are

0.006	0.9	0.03	30	16	21	Total = 68

A further refinement in the procedure, one that takes into account the non-planar character of the trajectory, reduces corrections 5 and 6 to insignificant size.

Consider Figure 2 -- an edge view of the trajectory. In this picture a planar path appears as a straight line. The procedure until now has been to project $\underline{r}_o$ ahead to perigee $\underline{r}_{p\,1}$ and then to determine $\underline{r}_L$ to satisfy the time constraint. The desired perigee has then been chosen to be $\underline{r}_{p\,4}$, in the plane of $\underline{r}_o$ and $\underline{r}_L$. The technique has been to change perigee from $\underline{r}_{p\,1}$ to $\underline{r}_{p\,4}$, a procedure which is valid only if the trajectory is sufficiently planar.

A correct procedure would be to project $\underline{r}_{p1}$ ahead to $\underline{r}'_L$, the position of the spacecraft at the time the landing site is at $\underline{r}_L$. Then, the correction should involve $\underline{r}'_L$ and $\underline{r}_L$.

An equivalent method would be to locate the point $\underline{r}_{p\,2}$, the perigee that the spacecraft would have to achieve in order to rendezvous with the landing site at $\underline{r}_L$. Then, $\underline{r}_{p\,1}$ and $\underline{r}_{p\,2}$ should be used for $\underline{r}'_p$ and $\underline{r}_p$ in Step 3 of perigee guidance.

Instead of either of the above correct procedures, the following approximate method has been found to be quite satisfactory. Determine $\underline{r}_{p\,3}$, the perigee in the plane of $\underline{r}_o$ and $\underline{r}'_L$ having the same central angle from $\underline{r}_o$ as $\underline{r}_{p\,1}$ and with magnitude $r_{p\,1}$. Then use $\underline{r}_{p\,3}$ and $\underline{r}_{p\,4}$ for $\underline{r}'_p$ and $\underline{r}_p$.

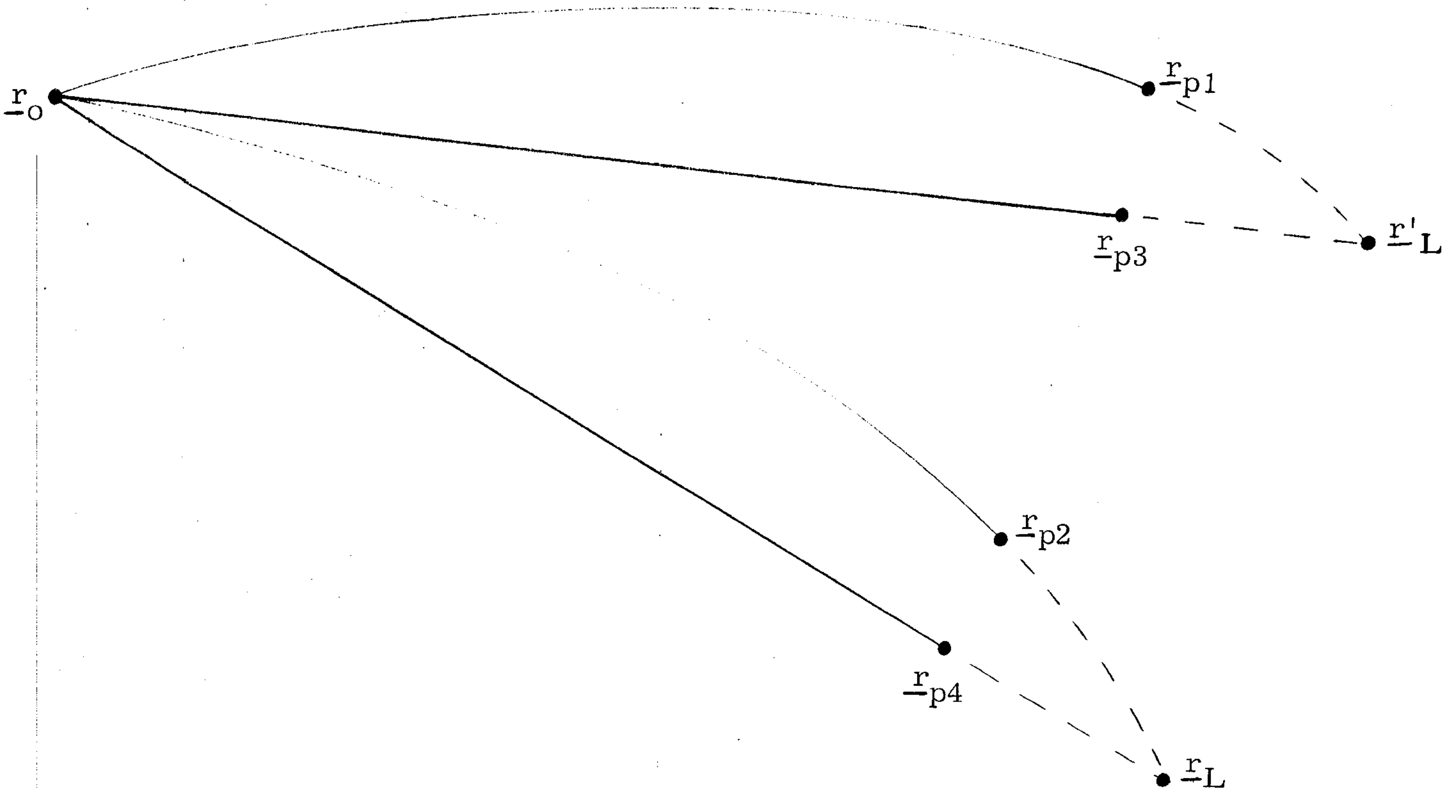
The calculations for the improved perigee guidance are now as follows:

1) a. Calculate t, r'_p, $\underline{r}'_p$, and $\cos\theta$.

Step 1 of perigee guidance.

b. Estimate the time of arrival at perigee.

$$E = \cos^{-1}\left(\frac{1 - \frac{r}{a}}{e}\right)$$



$$t_p = t + \sqrt{\frac{a^3}{\mu}} (E - e \sin E) + 16 \times 10^{-10} r_o (r_p - r'_p)$$

2) a. Locate $\underline{r}_L$.

Step 2a of perigee guidance.

b. Calculate the new $\underline{r}'_p$ ($\underline{r}_{p3}$ of Figure 2).

$$\underline{r}'_L = \underline{r}'_p \cos \alpha + r'_p \, \underline{u}(\underline{r} \times \underline{r}'_p) \times \underline{r}'_p \sin \alpha$$

$$\underline{\eta}' = \pm \underline{u}_{\underline{r}_o} \times \underline{r}'_L$$

Step 2 of perilune guidance with $\underline{\eta}'$ and $\underline{r}'_p$.

c. Determine $\underline{r}_p$.

Step 2 of perilune guidance.

3) Calculate the conic velocities.

Step 3 of perilune guidance.

4) Difference the conic velocities.

Step 4 of perilune guidance.

With improved perigee guidance the velocity corrections under perfect injection and observations are

0.006 0.9 0.03 42 1 4 Total = 48

Although corrections 5 and 6 have been practically eliminated, number 4 has become even larger.

In an attempt to reduce correction 4, the velocity correction at point 3 was changed from perilune guidance to position guidance, the target being point 4. Note that this cannot be done at point 2 because of the resulting danger of crashing into the moon. It is necessary to establish perilune at point 2. At point 3, within two hours of perilune, a large velocity change is required to substantially alter perilune distance; hence, it is safe to ignore perilune and aim for point 4 provided the velocity correction is not excessively large.

Using this last method, the six velocity corrections under ideal conditions are

0.006 0.9 7 2 0.3 4 Total = 14

8. Computer Simulation

Although the last procedure provides a great improvement in the perfect case, this is not always true in the more realistic non-ideal situation. It appears that the decision as to the type of correction to be applied at point 3 should be made at that point and not in advance. This idea will be discussed in more detail after the presentation of data.

The following four methods of circumlunar guidance were considered:

1) Original - the guidance procedure in Section 7.

2) Adjust time - *original with perigee guidance containing the more accurate estimate of perigee time of arrival.*

3) Improved perigee - original with improved perigee guidance.

4) At point 3 aim for point 4 - original with improved perigee guidance and position guidance at point 3.

For each guidance procedure, the results of five Monte Carlo runs are presented below. The runs within a set all have different random initial deviations and uncertainties generated from the RMS values. Navigation and control errors are also randomly produced. However, the random numbers used for each set of runs are the same, thereby making equivalent runs comparable. The RMS errors are given in Table 1. Tables 2 through 5 show the magnitudes of the required velocity corrections. The conditions that occur at indicated perigee, i.e., when the AGC indicates that the spacecraft is at vacuum perigee, are given in Tables 6 through 9.

Comparison of the data in Tables 4 and 8 with that in Tables 5 and 9 indicates that the following method for guiding at point 3 may be valid.

1) Calculate the position guidance velocity correction to arrive at point 4.

2) Make this correction if it is less than some value (maybe 20 miles per hour).

3) If it is too large, apply perilune guidance.

Another good reason for not making too large a position guidance type of velocity correction at point 3 is that the probability of crashing into the moon is kept small.

INITIAL ERRORS

		Altitude	Range	Track
Position (miles)	Deviations	11.92	2.88	.74
	Uncertainties	.96	1.44	.37
Velocity (miles per hour)	Deviations	15.62	8.88	10.2
	Uncertainties	7.81	4.44	5.1

SYSTEM ERRORS

Sextant	10 seconds
Earth horizon	1 mile
Moon horizon	1/2 mile
Velocity correction	
Magnitude	.1%
Angle	.1%
Velocity correction uncertainty	
Magnitude	.01%
Angle	.01%

TABLE 1 RMS ERRORS

RUN NUMBER

VELOCITY CORRECTIONS (miles per hour)	1	2	3	4	5
1	74	80	17	130	51
2	13	9	2	3	6
3	5	1	1	1	1
4	57	33	15	43	33
5	48	69	37	81	10
6	34	26	20	29	25
TOTAL	231	218	93	287	127

TABLE 2

VELOCITY CORRECTIONS FOR GUIDANCE PROCEDURE 1 - ORIGINAL

RUN NUMBER

VELOCITY CORRECTIONS (miles per hour)	1	2	3	4	5
1	74	80	17	130	51
2	13	9	2	3	6
3	5	1	1	1	1
4	74	46	23	62	31
5	17	17	15	19	15
6	28	26	19	27	23
TOTAL	210	178	78	242	127

TABLE 3

VELOCITY CORRECTIONS FOR GUIDANCE
PROCEDURE 2 - ADJUST TIME

RUN NUMBER

VELOCITY CORRECTIONS (miles per hour)	1	2	3	4	5
1	74	80	17	130	51
2	13	9	2	3	6
3	5	1	1	1	1
4	88	56	33	73	43
5	2	1	1	2	1
6	3	1	1	1	1
TOTAL	185	148	56	210	103

TABLE 4

VELOCITY CORRECTIONS FOR GUIDANCE
PROCEDURE 3 - IMPROVED PERIGEE

RUN NUMBER

VELOCITY CORRECTIONS (miles per hour)	1	2	3	4	5
1	74	80	17	130	51
2	13	9	2	3	6
3	8	37	8	46	25
4	10	35	3	41	19
5	1	1	1	1	1
6	6	4	2	7	7
TOTAL	112	165	33	228	109

TABLE 5

VELOCITY CORRECTIONS FOR GUIDANCE
PROCEDURE 4 - AT 3 AIM FOR 4

RUN NUMBER

	1	2	3	4	5
Change in perigee time (hours)	.35	.11	.13	.15	.24
Change in perigee radius (miles)	2.0	3.7	1.0	.7	2.2
Actual radial velocity (miles per hour)	64	54	31	22	37
Error in heading to target (degrees)	.07	.08	.05	.06	.07
Change in range perigee to target (miles)	506	293	272	317	382
Change in inclination angle (degrees)	-4.1	-2.3	-2.0	-2.9	-2.5
Change in perigee latitude (degrees)	-.4	+.1	+.2	-.2	+.2
Change in perigee longtitude(degrees)	-7.5	-4.8	-4.5	-5.3	-6.3

TABLE 6

TERMINAL CONDITIONS AT INDICATED PERIGEE FOR GUIDANCE PROCEDURE 1 - ORIGINAL

RUN NUMBER

	1	2	3	4	5
Change in perigee time (hours)	.35	.11	.13	.15	.24
Change in perigee radius (miles)	2.0	3.7	1.0	.7	2.2
Actual radial velocity (miles per hour)	64	55	31	21	37
Error in heading to target (degrees)	.07	.08	.04	.06	.07
Change in range perigee to target (miles)	506	292	272	315	382
Change in inclination angle (degrees)	-3.8	-1.6	-1.7	-2.1	-2.6
Change in perigee latitude (degrees)	-.2	+.5	+.3	+.3	+.2
Change in perigee longtitude (degrees)	-8.4	-4.7	-4.4	-5.1	-6.3

TABLE 7

TERMINAL CONDITIONS AT INDICATED PERIGEE FOR GUIDANCE PROCEDURE 2 - ADJUST TIME

RUN NUMBER

	1	2	3	4	5
Change in perigee time (hours)	.35	.11	.14	.15	.24
Change in perigee radius (miles)	2.0	3.7	1.0	.7	2.2
Actual radial velocity (miles per hour)	64	52	31	20	37
Error in heading to target (degrees)	.02	.03	.006	.01	.02
Change in range perigee to target (miles)	342	126	110	150	219
Change in inclination angle (degrees)	-4.2	-1.8	-1.9	-2.3	-2.9
Change in perigee latitude (degrees)	-1.2	-.4	-.5	-.6	-.8
Change in perigee longtitude (degrees)	-6.1	-2.3	-2.1	-2.8	-3.9

TABLE 8

TERMINAL CONDITIONS AT INDICATED PERIGEE FOR GUIDANCE PROCEDURE 3 - IMPROVED PERIGEE

RUN NUMBER

	1	2	3	4	5
Change in perigee time (hours)	.02	.21	.03	.24	.12
Change in perigee radius (miles)	2.1	3.8	1.1	.9	1.5
Actual radial velocity (miles per hour)	59	63	30	27	22
Error in heading to target (degrees)	.02	.03	.007	.01	.02
Change in range perigee to target (miles)	36	199	10	214	94
Change in inclination angle (degrees)	-.3	-2.5	-.3	-2.9	+1.8
Change in perigee latitude (degrees)	0	-.6	-.1	-.7	+.5
Change in perigee longtitude (degrees)	-.6	-3.6	-.2	-3.9	+1.8

TABLE 9

TERMINAL CONDITIONS AT INDICATED PERIGEE FOR GUIDANCE PROCEDURE 4 - AT 3 AIM FOR 4