Massachusetts Institute of Technology
Instrumentation Laboratory
Cambridge, Massachusetts

Space Guidance Analysis Memo #25-64

TO: SGA Distribution
FROM: Gerald M. Levine
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SUBJECT: Midcourse Monte Carlo Simulation Model

This memo describes briefly the procedures and data currently being used for Monte Carlo simulation of the midcourse guidance and navigation system. Because of the large number of computer runs now under consideration, a review of the model described below should be made at this time. All comments and suggested changes will be gratefully accepted.

The simulation is performed by integrating two trajectories: the actual and the estimated. A fixed schedule of velocity corrections and star-horizon observations is assumed. The schedule is adjusted, however, by the estimated deviation in time-of-arrival at the target.

At the correction times, impulsive velocity changes are added to both the actual and estimated velocities. The estimated position and velocity vectors are updated when an observation is made.

Errors are introduced into the system as follows:

I. Injection

Let the nominal trajectory be defined by the six-dimensional initial state vector

$$\begin{pmatrix} \underline{r}_{NOM_0} \\ \underline{v}_{NOM_0} \end{pmatrix}$$

To generate actual initial conditions, use is made of the covariance matrix of injection deviations X_0.

As in SGA Memo #56, let

$\underline{\ell}_i$ = Eigenvalues of X_0 matrix($i = 1, 2, \ldots, 6$)

$\underline{x}_i$ = Eigenvectors of X_0 matrix($i = 1, 2, \ldots, 6$)

Then, the initial state vector is given by

$$\begin{pmatrix} \underline{r}_0 \\ \underline{v}_0 \end{pmatrix} = \begin{pmatrix} \underline{r}_{NOM_0} \\ \underline{v}_{NOM_0} \end{pmatrix} + \sum_{i=1}^{6} \text{RNDMN}\left(\sqrt{\ell_i}\right)\underline{x}_i$$

If E_0 is the covariance matrix of injection uncertainties, then the estimated initial state vector is obtained in the same manner.

$$\begin{pmatrix} \underline{r}_{EST_0} \\ \underline{v}_{EST_0} \end{pmatrix} = \begin{pmatrix} \underline{r}_0 \\ \underline{v}_0 \end{pmatrix} + \sum_{i=1}^{6} \text{RNDMN}\left(\sqrt{\lambda_i}\right)\underline{e}_i$$

where the λ_i and $\underline{e}_i$ are the eigenvalues and eigenvectors of the E_0 matrix.

A third matrix that is used is the assumed covariance matrix of injection uncertainties E_0^*. This is the matrix (actually, its square root) which is used to calculate the weighting vectors for measurement incorporation. (See SGA Memo #40.)

The matrices X_0, E_0, and E_0^* are presently taken to be equal, even though they are three different matrices and could have different values.

The error matrices used for translunar and transearth injection are complete 6×6 matrices including cross correlation terms. The square roots of the diagonal elements of the injection error matrices in local vertical coordinates are shown in Table 1. The correlations are not shown in the table.

II. Velocity Corrections

Let

$\Delta \underline{v}_D$ = *Desired velocity correction calculated from* $\underline{r}_{EST}$ and $\underline{v}_{EST}$.

$\Delta \underline{v}_M$ = Measured velocity correction.

$\Delta \underline{v}$ = Actual velocity correction.

$\Delta \underline{v}_M - \Delta \underline{v}_D$ = Deviation in velocity correction.

$\Delta \underline{v} - \Delta \underline{v}_M$ = Uncertainty in velocity correction.

Both the uncertainty and the deviation in a velocity correction are assumed to be composed of a normally distributed angular error and a rectangularly distributed magnitude error. The statistics of the velocity correction errors are given in Table 2.

III. Star-Horizon Observations

Required in the calculation of weighting vectors for measurement incorporation is the assumed variance of the measurement error. This is the sum of the assumed variances of the sextant error and the horizon error of the planet involved in the measurement. The actual errors are generated from the following model:

$$\alpha_n = \alpha_{n-1} e^{-\lambda(t_n - t_{n-1})} + \text{BIAS} + \text{RNDMN}(\sigma)$$

where α_n and α_{n-1} are the measurement errors at the present and previous times t_n and t_{n-1}, λ is the sextant correlation coefficient, and σ is the square root of the sum of the actual variances of the sextant and horizon. The values of the various parameters are shown in Table 3.

IV. Physical Constants

All physical constants are presently assumed to be perfectly known, but provisions exist for inserting errors in the gravitational constants of the earth, moon and sun, and in the radii of the earth and moon.

		Altitude	Range	Track
Translunar Injection	Position (miles)	0.43	2.32	0.35
	Velocity (ft/sec)	16.01	3.39	8.53
Transearth Injection	Position (miles)	0.07	0.48	0.10
	Velocity (ft/sec)	3.05	1.18	2.68

Table 1. RMS Injection Errors

		RMS Angular Error (Rad)	Maximum Magnitude Error (ft/sec)
	Uncertainties	0.01	0.2
Translunar Deviations	$\Delta V \geq 4$ ft/sec	0.04	1.3
	$\Delta V < 4$ ft/sec	0	0
Transearth Deviations	$\Delta V \geq 15$ ft/sec	0.01	5.6
	$\Delta V < 15$ ft/sec	0	0

Table 2. Velocity Correction Error Statistics

	Actual	Assumed
1σ Earth Horizon Error (miles)	1	1
1σ Moon Horizon Error (miles)	$\frac{1}{2}$	$\frac{1}{2}$
1σ Sextant Error (sec)	10	10
Sextant Bias (sec)	0	-
Sextant Correlation Parameter (hours^{-1})	0	-

Table 3. Observation Error Statistics