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SGA Memo #10

TO: SGA Distribution

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SUBJECT: b-Vectors for Angle and Angle Rate Measurements in Arbitrary Coordinate System

This memo presents b-Vectors for angle and angle rate measurements taken in a measurement coordinate system (M-frame) and used to compute estimates of a state vector represented in a different system (e. g. platform frame-P frame).

The measurement gimbal system is that of the LEM rendezvous radar which tracks the CSM to obtain relative position ($\underline{R}$) and velocity ($\underline{V}$) of the CSM with respect to the LEM. The outer gimbal reads elevation angle (β) and the inner gimbal reads azimuth angle (θ). The geometry is illustrated in Fig. 1. The b-Vector for a given measurement is in general a 6-dimensional vector which may be represented as

$$\underline{b} = \begin{bmatrix} \underline{b}_0 \\ \underline{b}_3 \end{bmatrix} \text{ where } \underline{b}_0,\ \underline{b}_3 \text{ are 3 x 1 column vectors}$$

and which satisfies

$$\delta Q = \underline{b}^T \delta \underline{x}$$

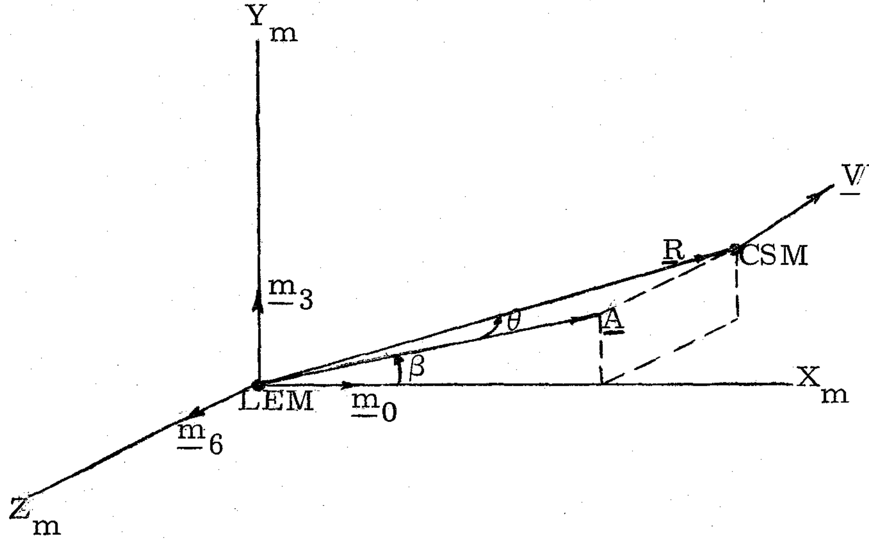
where δQ = deviation in measured quantity

$$\delta \underline{x} = \begin{bmatrix} \delta \underline{r} \\ \delta \underline{v} \end{bmatrix} = \text{6-dimensional state deviation vector with coordinates in P-frame.}$$

The vector $\delta \underline{x}$ represents deviations in position and velocity of the vehicle containing the tracking unit (the LEM in this case).

The rotation matrix which transforms vectors with P frame coordinates to M-frame coordinates is $\overset{*}{M}_{p-m}$. The three row vectors of this matrix are $\underline{m}_0$, $\underline{m}_3$, and $\underline{m}_6$, which are the three unit vectors representing the M-frame.

The b-Vectors corresponding to the two angle measurements (β, θ) and two angle rate measurements ($\dot{\beta}$, $\dot{\theta}$) are tabulated in Table I. All vectors are expressed in P-frame coordinates.



$$\underline{A} = (\underline{R} \cdot \underline{m}_0)\,\underline{m}_0 + (\underline{R} \cdot \underline{m}_3)\,\underline{m}_3$$

$$A = |\underline{A}|\ ,\quad R = |\underline{R}|$$

Fig. 1. Measurement frame geometry

Table I

Measurement	b-Vector (6 × 1)	
	$\underline{b}_0$ (3 × 1)	$\underline{b}_1$ (3 × 1)
Elevation angle (β)	$(\underline{R} \times \underline{m}_6)/A^2$	$\underline{0}$
Azimuth angle (θ)	$((\underline{R} \times \underline{m}_6) \times \underline{R})/R^2A$	$\underline{0}$
Elevation angle rate ($\dot{\beta}$)	$(\underline{m}_6 \times \underline{V} + 2\dot{\beta}\underline{A})/A^2$	$(\underline{R} \times \underline{m}_6)/A^2$
Azimuth angle rate ($\dot{\theta}$)	$1/R^2A\ [\,\underline{V} \times (\underline{R} \times \underline{m}_6) + \underline{P}_6 \times (\underline{R} \times \underline{V}) + \dot{\theta}(R^2\,\underline{A} + 2A^2\,\underline{R})/A]$	$((\underline{R} \times \underline{m}_6) \times \underline{R})/R^2A$