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Space Guidance Analysis Memo #4-66

TO: SGA Distribution

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SUBJECT: Controlling the Attitude and Attitude Rate of the LEM with the Gimballed Descent Propulsion System.

Introduction

The personnel working on the LEM digital autopilot have just completed an LGC program for steering the attitude and attitude rate of the spacecraft with the descent engine trim gimbal. This venture represents somewhat of a departure from our previous philosophy of merely using the trim gimbal to null the disturbing acceleration due to the DPS moment offset. While merely nulling the moment offset with the trim gimbal had the advantage of simplicity in the control law, this philosophy does not appear consistent with the extremely tight RCS propellant budget contained in the recently negotiated Performance and Interface Specification.

This memo is intended to bring interested personnel up to date on the new philosophy of trim gimbal control and the current status of the implementation of the new philosophy.

Background

For some time it has been known that attitude hold could be performed by the DPS trim gimbal. Thus, the trim gimbal can cause the LEM to limit cycle inside the RCS jet dead band. (See Figure 1.) The LEM spacecraft designers originally intended the trim gimbal merely to be used as a moment offset control. By keeping the average moment offset at or near zero foot-pounds the RCS jets would have to fire very rarely in the attitude hold kind of situation. In the more ambitious kind of trim gimbal control we find a control law which actually steers the spacecraft, actuating the trim gimbal on the basis of vehicle attitude error and rate as well as disturbing acceleration; and the jets do not fire at all in attitude hold. This kind of control views the angular acceleration due to DPS engine thrust not merely as a disturbance but more constructively as a control agency.

The gimballed DPS engine can actually impress very large torques on the LEM, on the order of 10 times the torque due to the firing of a single RCS jet. (See Figure 2.) The advantage of the RCS jets for the control function is that they generate their torques very quickly upon receiving a command. When a quiescent jet is commanded to thrust, the delay before full thrust and torque is achieved is only a matter of milliseconds. On the other hand, if the thrust vector of the DPS engine passes through the c.g., approximately 30 seconds are required before the maximum lever arm of the moment can be achieved by gimballing the DPS engine ! (The gimbal angle changes at the rate of 0.2^{o}/sec.) Another 30 seconds are required to return the gimbal to the zero moment arm position.

An interesting way of comparing the speed of the RCS jets and the speed of the trim gimbal is to compute how many milliseconds are required for the trim gimbal to generate a torque impulse equivalent to the minimum impulse generated by a single RCS jet. A single RCS jet generates a minimum impulse of approximately 4 ft-lb-seconds; and approximately 15 milliseconds are required to generate this torque impulse. On the other hand, about 400 milliseconds are required to generate 4 ft-lb-seconds with the trim gimbal.

The slowness of the trim gimbal quite evidently makes it unsuitable for the control function when the steering commands are changing rapidly. Thus, during phase 2 (the visibility phase) when the radar data is being assimilated in the navigation and guidance system (causing the steering errors to fluctuate a great deal) and the astronaut is causing site re-designations, a trim gimbal law which attempts to steer the spacecraft is of marginal or even negligible utility.

But during the major portion of the first phase of the landing trajectory, when the steering commands change in a smooth and gradual manner, the trim gimbal should be fast enough to steer the LEM without the assistance of RCS jet firings.

Because the RCS propellant budget for phases 1 and 2 of the landing are so tight (in fact, the budget is partially predicated on a trim gimbal steering technique) it appears necessary to try to exploit the ability of the trim gimbal to steer, at least during phase 1.

Because the trim gimbal is so slow, a control law which does not add to the slowness in reacting to attitude errors is required if the RCS jets are not going to have to fire. The faster the trim gimbal control law the better. A time-optimal system of control is the most desirable kind of control here. There is no penalty connected with running the trim gimbal drive. Therefore, a time-optimal control is the correct pay-off function.

The Time-Optimal Control Law

The following time-optimal control law has been derived by W.S. Widnall as an exercise in Professor Athan's class and by Bard Crawford, Dick Goss and George Cherry working from Athan's notes. SGA Memo # 3-66 contains a detailed derivation of this law.

$$K = FLR/I$$

$$\Delta = -\,\mathrm{sgn}\,(K\dot{\theta} + \ddot{\theta}|\ddot{\theta}|/2)$$

$$u = -\,\mathrm{sgn}\,[K^2(\theta-\theta_D) + \ddot{\theta}^3/3 - \Delta K\dot{\theta}\,\ddot{\theta} - \Delta(-\Delta K\dot{\theta} + \ddot{\theta}^2/2)^{3/2}$$

where θ is the measured angle around a controlled axis and θ_D is the desired orientation around that axis. The quantities $\dot{\theta}$ and $\ddot{\theta}$ are simply the rate and acceleration around the axis. This control law drives $\theta - \theta_D$, $\dot{\theta}$, and $\ddot{\theta}$ to zero in minimum time. The quantity K is computed every second (in the guidance equations) because it changes very slowly.

This control law has been simulated in a MAC program written by Dick Goss. Figures 3a and 3b illustrate the convergence of the vehicle's state to a tight limit cycle about the origin of the $\theta - \theta_D$, $\dot{\theta}$, $\ddot{\theta}$ phase space. The MAC simulation which produced the data plotted on figures 3a and 3b used a sampling frequency of 5 cycles per second and a first order lag of 0.1 second time constant on the trim gimbal drive rate. The Kalman filter was not simulated. (The state variables used in the control law simulation were perfect; in the real system the state variables must be inferred from the CDU measurements by a Kalman filter.)

It is interesting to compute how large an initial deviation, δ_0, (see figure 2 for the definition of δ) can be handled during the initial DPS start-up without any RCS jet firings. Assume that the RCS jets have established the following conditions just prior to DPS engine ignition.

$$\theta - \theta_D = 0$$
$$\dot{\theta} = 0$$

Then the maximum angular error which will be encountered (without any jet firings) is

$$(\theta - \theta_D)_{max} = \frac{2}{3} \frac{F L}{I} \frac{\delta_0^3}{\dot{\delta}^2}$$

Assuming the following realistic numbers

$F = 3000$ lbs

$L = 3$ ft.

$I = 22{,}000$ ft-lb-sec^2

$\dot{\delta} = 0.2^{\circ}$/sec.

we obtain

$$(\theta - \theta_D)_{max} = \frac{300}{44} \delta_0^3 \quad \text{(degrees)}$$

Now, assuming that we will tolerate an error of 2° without firing RCS jets,

$$(\theta - \theta_D)_{max} = 2^{\circ}$$

we can solve the above equation for δ_0

$$\delta_0 \approx 2/3 \text{ degree}$$

Thus, if we tolerate an attitude error of 2° (do not fire jets for smaller attitude errors) we can handle initial moment offsets of 2/3 degree and smaller without firing jets at all during the initial phase of the lunar landing.

Two-thirds of a degree happens to be the one sigma offset due to c.g. uncertainties and engine alignment uncertainties. Thus, with an initial phase of 3000 lbs thrust the one sigma case may be handled without any jet firings during almost all of phase 1. Obviously, with a lower initial thrust setting, say F = 1000 lbs, a larger initial δ_0 can be handled without exceeding an attitude error of 2^o and without firing RCS jets. In the practical case then, it seems fairly reasonable to expect very few RCS jet firings during the first 300 or so seconds of the DPS burn. Optimism must be guarded, however, until a complete LGC simulation of the Kalman filter and time-optimal control law is completed.

The Problem of Obtaining Accurate Switching

The trim gimbal control equation exhibited on page 3 results in a bang-bang type of control. The trim-gimbal drive command, u, is either plus or minus. The basis of the law is a surface whose equation is

$$\Delta = - \text{sign} (\dot{\theta} + \ddot{\theta} \, | \ddot{\theta} | /2K)$$

$$f (\theta - \theta_D, \dot{\theta}, \ddot{\theta}) = 0$$

where

$$f(\theta - \theta_D, \dot{\theta}, \ddot{\theta}) = \theta - \theta_D - \Delta\dot{\theta}\ddot{\theta}/K + \ddot{\theta}^3/3K^2 - \Delta(-\Delta K\dot{\theta} + \ddot{\theta}^2/2)^{3/2}/K^2$$

This is a surface in the $\theta - \theta_D$, $\dot{\theta}$, $\ddot{\theta}$ phase space which divides three space into two simply connected sub-spaces. On one side of the surface, + control is always applied; on the other side, − control is applied. The surface itself is the locus of all trajectories which go to the origin with zero or one switch of u. It is easy to see that f is always positive in a simply connected subspace on one side of f = 0, the switching surface. Consider the partial derivative of f with respect to $(\theta - \theta_D)$.

$$\partial f/\partial (\theta - \theta_D) = 1$$

Thus, f is zero on the surface, and positive if $\theta - \theta_D$ is incremented without changing $\dot{\theta}$ or $\ddot{\theta}$. Thus, if a given phase point lies in the surface, $(\theta - \theta_D)$ is

$$\theta - \theta_D = \Delta\dot{\theta}\,\ddot{\theta}/K - \ddot{\theta}^3/3K^2 + \Delta(-\Delta K \, \dot{\theta} + \ddot{\theta}^2/2)^{3/2}/K^2$$

If $\theta - \theta_D$ is "above" the surface

$$\theta - \theta_D > \Delta\dot{\theta}\ddot{\theta}/K - \ddot{\theta}^3/3K^2 + \Delta(-\Delta K\dot{\theta} + \ddot{\theta}^2/2)^{3/2}/K^2$$

and

$$f(\theta - \theta_D, \dot{\theta}, \ddot{\theta}) > 0$$

Similarly if $\theta - \theta_D$ is "below" the surface

$$f(\theta - \theta_D, \dot{\theta}, \ddot{\theta}) < 0$$

Since the phase trajectories which go to the origin lie in the surface and the trajectories which go near the origin lie near the surface, the sign of control which causes the phase trajectory to approach the surface must be reversed at nearly the instant of impinging on or piercing through the surface. (The case of being exactly on the surface need not be considered because the volume of the surface is zero, i.e., the points which lie on the surface comprise a set of measure zero). The proper control is evidently

$$u = -\operatorname{sgn}[f(\theta - \theta_D, \dot{\theta}, \ddot{\theta})]$$

since, for example, if f is positive $\theta - \theta_D$ is above the surface and the phase point can be forced toward the surface by negative control only.

The above control law is the perfect time optimum control system if the trim gimbal rate can be switched from one sign to the other without lag and if the signum functions and the control law are computed continuously. The trim gimbal does have dynamical lag, however. And the LEM DAP is a sampled data system; consequently, the control law is only computed periodically. The highest sampling frequency which appears computationally feasible is 5 cycles/second. (A higher sampling frequency causes the DAP to use more than its share of the LEM computer time.)

Let us consider first the effect of sampling when using the above control law. If the velocity of the phase trajectory is high relative to the speed of sampling, i.e., the phase point moves quite far during a sampling period, the trajectory may overshoot the surface considerably before the state is sampled and the trim gimbal reversal signal is sent to the gimbal drive. Relatively fast trajectories can result from either a high gain system (large $|\dddot{\theta}| = K$) or large initial conditions (large initial $\dot{\theta}$ and $\ddot{\theta}$).

When the velocity of the phase trajectory is high relative to the sampling frequency, convergence to the steady state limit cycle is slow. On the other hand, when the gain of the system is high relative to the sampling frequency, convergence to the limit cycle is slow; and, furthermore, the steady state limit cycle tends to have a very large amplitude. Both of these effects are due to the fact that the system trajectory tends to go rather far past the switching surface before the crossing is detected. A higher sampling rate would improve the situation - if a higher sampling frequency were possible.

The trim gimbal rate lag causes overshoots which cannot be compensated by increasing the sampling frequency. Something of the nature of prediction is required to compensate for the effect of the trim gimbal rate lag. Thus, the command to reverse the trim gimbal drive should be issued <u>before</u> the switching surface is reached.

There is a rigorous solution to this problem. The theory which produced the "time-optimal" control law exhibited in this memo and derived in Widnall's memo is not deficient. The difficulties arise because the true system has been simplified in order to obtain a fairly simple control algorithm. In SGA Memo # 3-66, Widnall assumes that the plant is defined by

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= \alpha \\ \dot{\alpha} &= uK \end{aligned}$$

where $u = \pm 1$ and K is the scale factor. This simplified model of the plant leads to a useful and fairly simple control law; but the difficulties referred to do crop up. A more realistic model of the plant would be

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= \alpha \\ \dot{\alpha} &= a\dot{\delta} \\ \ddot{\delta} &= b(\dot{\delta}_c - \dot{\delta}) \end{aligned}$$

Here, b is the reciprocal of the time constant of the trim gimbal rate response, $a = FL/I$, and $\dot{\delta}_c$ is $\pm 0.2^o/sec$. These equations can be put into a form suitable for the optimizing theory used in SGA Memo # 3-66 by rewriting them as

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = X_3$$
$$\dot{X}_3 = a'X_4$$
$$\dot{X}_4 = b(u - X_4)$$

where $u = \pm 1$, $a' = a/5$, and $X_4 = 5\dot{\delta}$.

Several difficulties crop up if this more realistic, more complicated model is used. First, the dimension of the state and the order of the system is increased to four; and the forth state variable, proportional to $\dot{\delta}$, must be estimated. Secondly, more complicated transcendental functions enter the resulting time-optimal control law. Finally, if the continuous theory is used, the effect of sampling is not taken into account by this model either.

Rather than make a rigorous appeal to the optimization theory, we have tried several fixes of an engineering nature.

The first method used to obtain accurate switching and small amplitude limit cycles was to predict the time-to-go to reach the surface. An algorithm for computing T_{go} exactly is very, very complicated. An approximate T_{go} can be computed from

$$T_{go} = - f / \dot{f}$$

where

$$\dot{f} = (u - \Delta)\left[\ddot{\theta}^2/K - \Delta\dot{\theta} - 3\Delta\ddot{\theta}\,(-\Delta K\,\dot{\theta} + \ddot{\theta}^2/2)^{1/2}/2K\right]$$

Since the switch should take place on the surface, i.e., when f = 0, the above calculation yields an estimate for T_{go}. Note that the derivative is zero (the trajectory moves parallel to the surface) when $u = \Delta$. The time of switch command should take the trim gimbal rate lag into account. Thus

$$T_s = T_{go} - 3\tau$$

is a useful expression for the time of switching, where τ is the trim gimbal lag time constant. This method of predicting the time of switching has been simulated on the MAC program written by Dick Goss. The results reveal fast convergence to a small amplitude limit cycle. But the logic is fairly complicated since the trim gimbal may actually reverse before the surface is reached. Then the decision must be made as to whether the system state should be considered close enough to the switching surface to consider the sign of the drive appropriate.

Another technique which has been investigated by simulation is the policy of using a coast prior to intercepting the surface. By turning off the drive before the surface is reached the velocity of the phase trajectory is decreased, reducing the overshoot which results from sampling. Furthermore, because the trim gimbal is stopped when the surface crossing is detected, the delay in reaching trim gimbal speed in the right direction is reduced by a factor of 1/2. Simulation of this technique reveals it to reach a small amplitude limit cycle. There are two techniques for establishing a coast prior to reaching the switching surface. One technique defines a second surface with smaller gain. Thus, when

$$f_c(\theta - \theta_D, \dot{\theta}, \ddot{\theta}, K < K_{true}) = 0$$

is encountered the trim gimbal drive is switched off. When the true surface is reached the control is turned on according to

$$f_s(\theta - \theta_D, \dot{\theta}, \ddot{\theta}, K = K_{true}) = 0$$

This could be mechanized by noting that the drive signal is applied when

$$\text{sign}(f_s) = \text{sign}(f_c)$$

Coasting is used when

$$\text{sign}(f_s) \neq \text{sign}(f_c)$$

The drawback of this mechanization is the need to compute two complicated functions, f_s and f_c.

Another way of providing a coast is to coast whenever

$$\text{sign}(\dot{f}) \neq \text{sign}(f)$$

This is easy to mechanize. The drawback is that the coasts may be quite long. Simulation of this shows it to work adequately.

Finally, a technique for reducing the amplitude of the limit cycle has been suggested by W.S. Widnall. His suggestion is to reduce the K used in the switching surface by a factor of about 0.5. The result of this gain reduction is to cause the system trajectory to approach the origin by switching back and forth across a surface which represents a lower gain system. Thus, in effect, the true system is made to behave like a lower gain system by switching the control signal sign back and forth. This policy results in a very small limit cycle. The angle limit on the steady state limit cycle is less than 0.1 degree. The settling time to attain this limit cycle is only a little longer than the truly optimum system. This policy is the one which has been programmed for the LGC and it is this policy which will be used unless the 100 % duty ratio on the trim gimbal drive turns out to be objectionable. If a smaller duty ratio should be required, a coasting policy using a reduced gain switching surface will be investigated.

Implementation of the Time-Optimal Control Law in the LGC

The time-optimal control law requires knowledge of the system state, θ, $\dot{\theta}$, and $\ddot{\theta}$. A Kalman filter has been programmed to estimate gimbal angles, gimbal angle rate, and gimbal angle acceleration. The Kalman filter is programmed in gimbal angle coordinates because the CDU measurements and the estimation procedure are performed four times for each calculation of the control law. The matrix multiplication required to resolve from gimbal angle coordinates to body coordinates is performed only when the control law is computed. The Kalman filter is described in R-499. The only thing to add to what is discussed in R-499 is that the components of the weighting vector are fitted as exponential functions which are then computed as simple recursion formulas. This mechanization of the $\underline{W}$ vector was suggested by Kurt Lanza. Suppose we can satisfactorily represent a Kalman filter weight by the following exponential functions of time

$$w(t) = a\, e^{-bt} + c$$

then

$$w(t+\Delta t) = a\, e^{-b(t+\Delta t)} + c$$

$$w(t+\Delta t) = e^{-b\Delta t}(a\, e^{-bt} + c) + c\,(1 - e^{-b\Delta t})$$

$$w(t+\Delta t) = e^{-b\Delta t}\, w(t) + c\,(1 - e^{-b\Delta t})$$

thus

$$w_n = \alpha\, w_{n-1} + \beta$$

is a recursion formula for w (t) where

$$\alpha = e^{-b\Delta t}$$

$$\beta = c\,(1 - e^{-b\Delta t})$$

Since Δt, the filter sampling time, is fixed, α and β are constants. Thus, only three constants, w_0, α, and β need be stored and the sequence of values of the w weight are handily generated by a simple multiplication and addition.

The Kalman filter computation is performed 20 times a second and takes 5 milliseconds a pass.

The control law computation (both axes) takes about 9 milliseconds. Thus, without violating the 14 millisecond maximum interrupt time, the control law can be performed with a filter calculation. If the control law is calculated 5 times per second, every fourth filter execution, the following computer load results

P axis	5 × 6 msec	=	30 msec
Filter	20 × 5 msec	=	100 msec
Control Law	5 × 9 msec	=	45 msec
Matrix Updates	2 × 3 msec	=	6 msec
			181 msec

Thus, about 18 % of the computer time is used by the DAP during the time the trim gimbal is used to steer the vehicle.

During each pass through the control law the angular error is checked. If the error exceeds a given bound (about 1 to 2 degrees) the jets are used for control until the vehicle's state is brought closer to the origin. Before exiting from the trim gimbal law to the RCS jet control law, the trim gimbal is set to drive $\ddot{\theta}$ to zero and a waitlist call to turn off the trim gimbal in

$$T = \ddot{\theta} \, / \, \dddot{\theta}$$

seconds is requested. When the jets have returned the spacecraft to the vicinity of the origin of the $\theta - \theta_D$, $\dot{\theta}$ phase plane, control is returned to the trim gimbal control law. The Kalman filter weights are re-initiated upon this return.

Figure 4 is a block diagram of the trim gimbal control law.

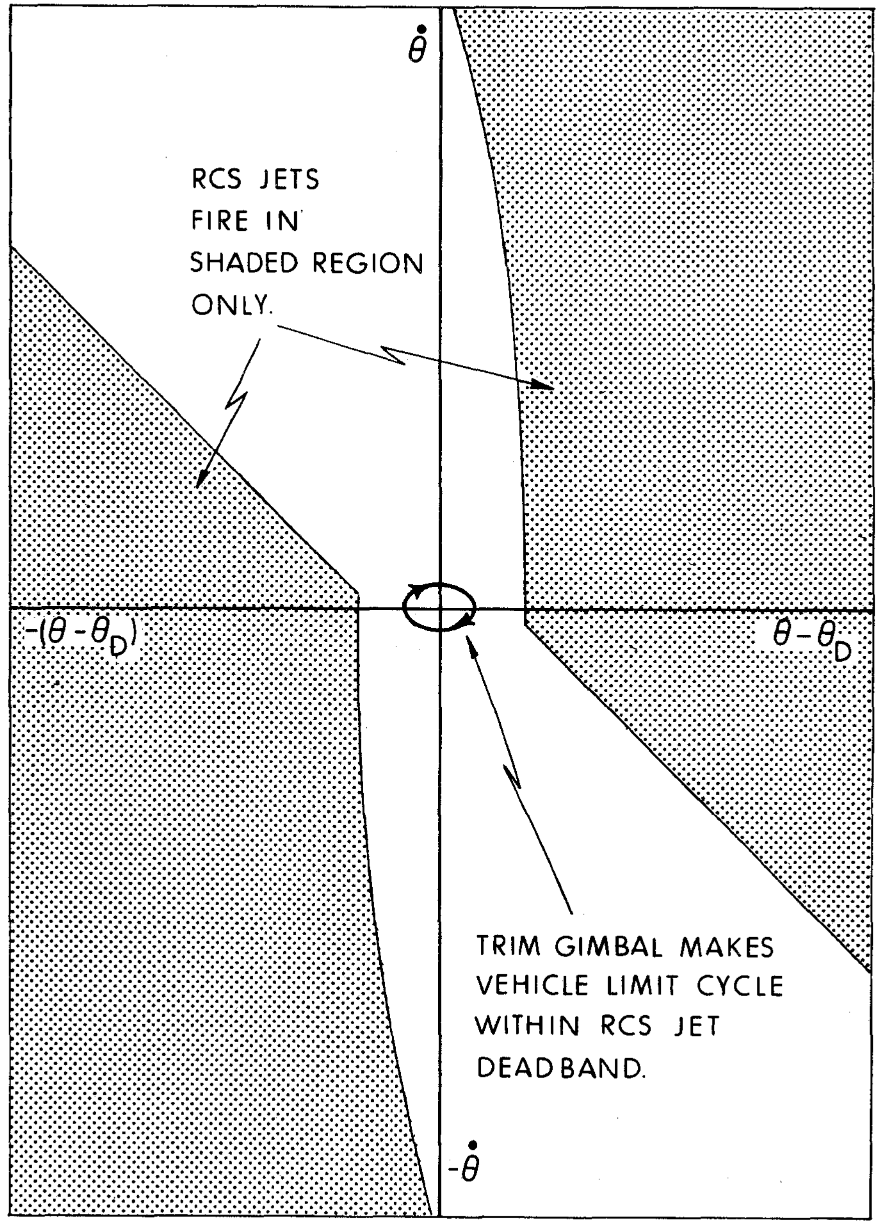
REFERENCES

1. Athans and Falb, " Optimal Control: An Introduction to the Theory and its Applications." Chapter 7 contains the relevant material on time-optimal control systems.

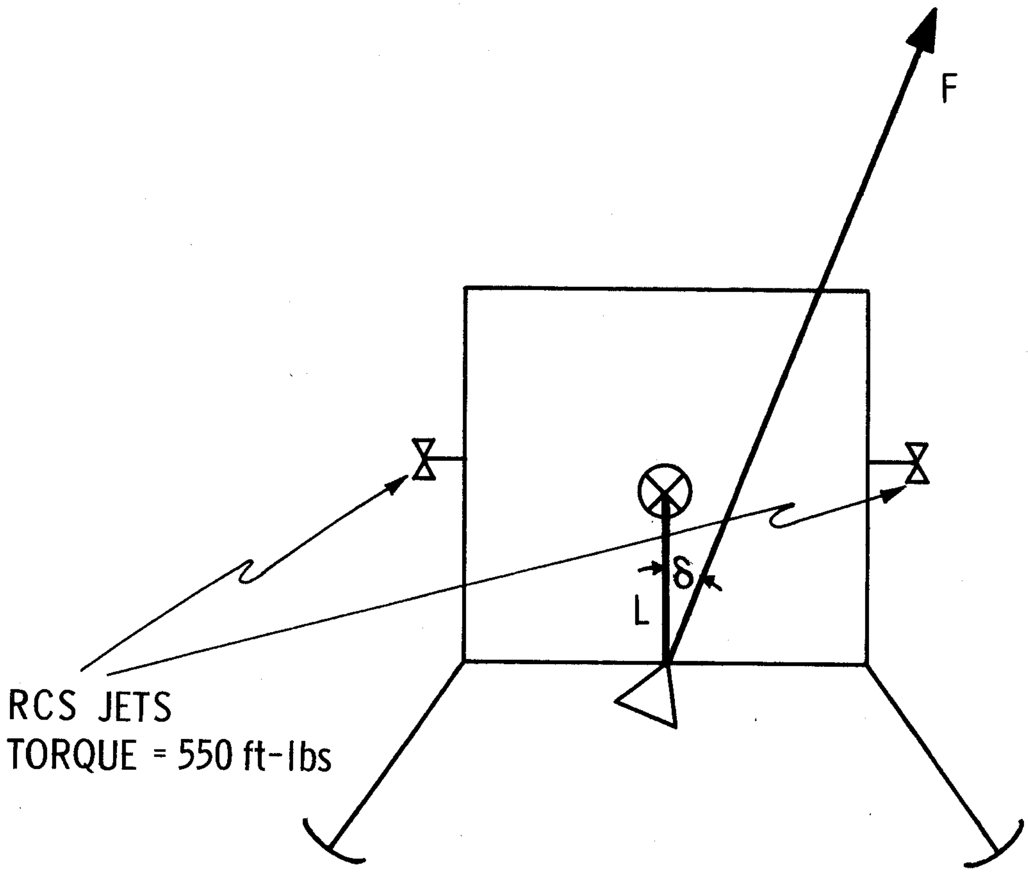
2. SGA Memo # 3-66. W.S. Widnall, Derivation of the Optimum Control Program for Steering the LEM Using the Gimballed Descent Engine.

3. Unnumbered memo to George Cherry from William S. Widnall, " Increasing the Stability of Non-Linear Bang-Bang Control Systems Through Gain Reduction. "

CONTROLLING THE LEM ATTITUDE AND ATTITUDE RATE WITH THE TRIM GIMBAL



LEM CONTROL TORQUE AGENTS



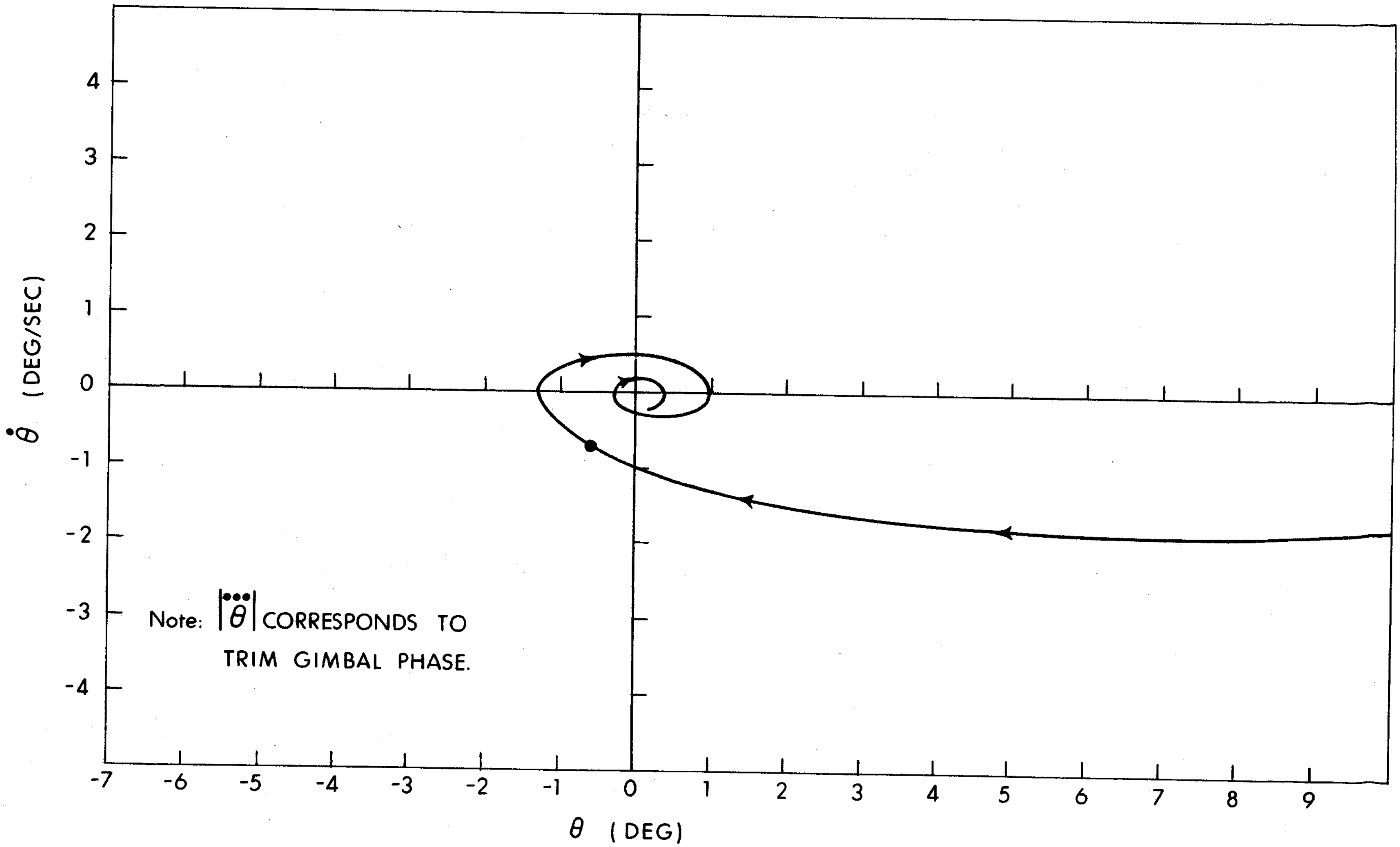
$$\ddot{\theta}_{TG} = \frac{FL\,\delta}{I}$$

$$\dddot{\theta}_{TG} = \frac{FL\,\dot{\delta}}{I}$$

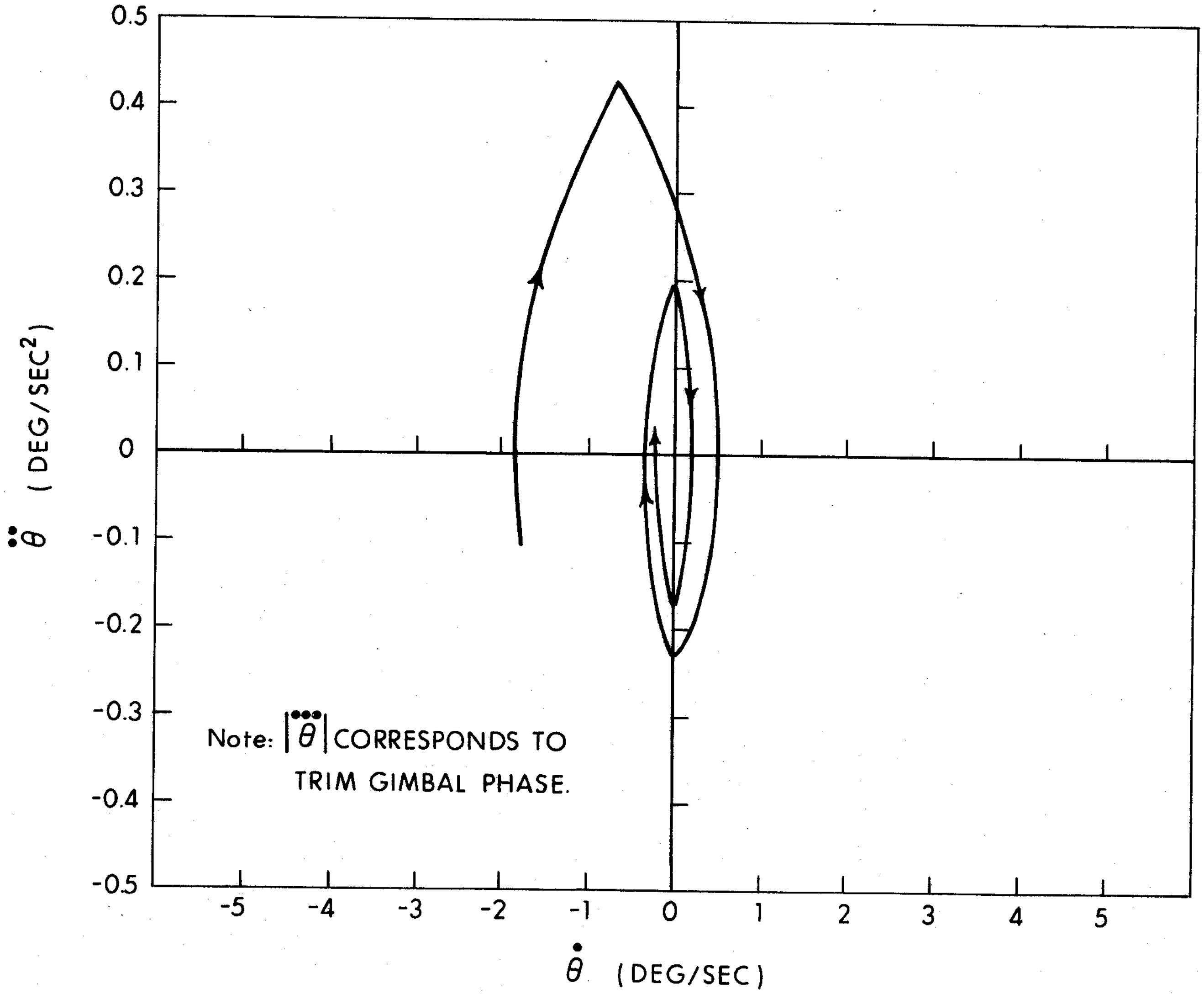
MAX. TORQUE FROM DPS ≈ 5000 ft-lbs (for $\delta = 6^{0}$)

$\dot{\delta} = \pm 0.2^{0}/\text{sec}$ MAXIMUM $\delta \approx 6^{0}$

$\dot{\theta}$ - θ PHASE PLANE PLOT ILLUSTRATING CONVERGENCE TO LIMIT CYCLE



$\ddot{\theta}$ - $\dot{\theta}$ PHASE PLANE PLOT ILLUSTRATING CONVERGENCE TO LIMIT CYCLE



TRIM GIMBAL CONTROL SYSTEM FOR Q AXIS