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Space Guidance Analysis Memo #2-64

TO: SGA Distribution
FROM: E. S. Muller, N. E. Sears
DATE: January 13, 1964
SUBJECT: Primary G&N Rendezvous Guidance Equations.

1. Introduction.

This memo summarizes the current guidance equations and technique for lunar orbit rendezvous. The rendezvous maneuver consists of two phases, midcourse and terminal. The midcourse rendezvous phase is initiated immediately after powered injection from surface launch or aborted landing maneuvers at typical ranges of 200 n. m. The objective of this phase is to establish a collision or intercept trajectory between the two vehicles by a series of velocity corrections initiated at the longest possible ranges. The terminal rendezvous phase controls the acceleration of the rendezvousing vehicle such that the relative velocity between the two vehicles is reduced to zero as the relative range decreases to a desired terminal separation distance. The terminal rendezvous phase typically starts at relative ranges of 5 n. m.

The guidance equations presented in this memo control both midcourse and terminal rendezvous maneuvers. The basic guidance and navigation technique is the same as that used during the translunar midcourse phases of the Apollo lunar mission. This guidance technique is described in Ref. 1, and was chosen for the rendezvous phases for the following reasons:

1) It is extremely flexible in that all ascent and abort trajectories (including CSM retrieval trajectories) can be handled with any valid tracking data provided by radar or optics.

2. Of the guidance techniques investigated, it provided the best performance in achieving effective velocity corrections at long ranges using the currently specified rendezvous radar, (Ref. 2). The first midcourse velocity correction is typically applied 5 to 10 minutes after ascent or abort injection, thereby limiting the required midcourse and terminal rendezvous propellant requirements.

3. Most of the guidance equations or subroutines required for this system exist in the AGC as programs required for other phases of the Apollo mission (translunar midcourse and orbital navigation phases).

There are two major differences between the guidance techniques for the rendezvous and translunar phases. The first concerns the input tracking or observation measurements. Tracking radar data between the two vehicles is used in the rendezvous phase rather than the optical star horizon or landmark measures used in the translunar navigation phase. For the specified rendezvous radar performance (Ref. 2), the tracking parameters that have proved most useful in the rendezvous phase are range rate ($\dot{R}$) and the two tracking angles measured with respect to the IMU. Other combinations of the six possible tracking parameters are possible, but result in higher ΔV requirements. This includes optical tracking angles which can be used as a back-up or in place of the tracking radar. The second difference between the guidance techniques for the rendezvous and translunar phases is the necessity of estimating tracking radar angle biases, for the current radar performance and installation tolerances in the long range mid-course rendezvous phase. The estimating technique is the same as that used for position and velocity deviations as described in section 2.

It should be noted that the guidance and navigation equations presented in the following sections are used in both the CSM and the LEM. Either vehicle could be the active vehicle controlling the rendezvous maneuver, and under normal operation, each would be solving the same mid-course and terminal rendezvous problem so that system operation could be monitored, and one guidance system take over in the case of indicated failure in the other. This type of operation, using the following guidance equations, is primarily used in the rendezvous phase of the mission. It should be pointed out, however, that this navigation technique is planned for all unpowered phases of the LEM mission. This includes the descent orbit phase in which the initial descent trajectory and perilune conditions are checked by both vehicles; and the LEM lunar surface phase in which the CSM orbit relative to the LEM landing site is determined by radar tracking on both vehicles in order to determine the launch trajectory aim point and timing.

2. General Comments.

The block diagram in Fig. 1 represents three major subdivisions of the mid-course and rendezvous guidance system. Each of these subdivisions will be considered separately. The guidance equations appropriate to each block will be presented along with the respective inputs and outputs necessary to interconnect the three blocks into an integrated system.

The basic notation used in the guidance equations is shown in Fig. 2. Some additional comment on this notation is appropriate here. All vectors are three-dimensional, except $\delta\underline{x}$, $\underline{e}$, $\underline{b}$ and $\underline{W}$, which are nine-dimensional. (It should be noted that letters representing vectors are underscored to distinguish them from statistical averages which have a bar above.) An extrapolated vector (or matrix), noted by a prime, is the value of the vector at time t_n computed from: first, the knowledge of its value at time t_{n-1}; second, the time elapsed $t_n - t_{n-1}$; and third, the equations governing its variation with time. The transpose of a vector appears in the equations as $\underline{A}^T$.

Some general definitions are given in Fig. 3. These definitions are consistent with those of Ref. 1. The only difference is the inclusion of measurement bias estimates ($\widehat{\underline{\text{B}}\text{IAS}}$) in the estimate of the state deviation vector ($\delta\hat{\underline{x}}$) with its associated error in the bias estimate ($\underline{\gamma}$). This results in the augmentation of the original covariance matrix, (noted as E_{LEM}) in Ref. 1, from a six by six matrix to a nine by nine matrix, and the augmentation of the original transition matrix, noted as Φ, from a six by six matrix to a nine by nine matrix (P). It can thus be seen, that the bias estimates are treated as additional state variables in the same manner as position ($\delta \underline{r}$) and velocity ($\delta \underline{v}$) deviations. It is necessary only to have a priori statistical knowledge of the biases to be estimated, and a knowledge of the manner in which the biases vary with time. This additional input data is represented by the bias covariance matrix $(E_{BIAS})_0$, and the bias transition matrix, Φ_{BIAS}. The bias estimate ($\hat{\underline{B}}IAS$) is a 3-dimensional vector, thus allowing for the estimate of three quantities in addition to the state deviation vectors ($\delta \underline{r}$ and $\delta \underline{v}$), e.g., the estimate of the bias in each of three independent measurements, or possibly the estimate of 3 Euler angles representing the platform or radar axes misalignment.

In the present system configuration, the error model chosen was one in which the measurements had constant biases. The initial bias covariance matrix is a diagonal matrix, each term on the diagonal being the mean square value (on an ensemble basis) of the bias in each of the three independent measurements used. The bias transition matrix becomes an identity matrix since the biases are constant. It should be stressed again that this error model is arbitrary. For example, if the biases were known to vary in some prescribed manner with time (e.g. linearly or exponentially), the only change required would be in the bias transition matrix.

The coordinate system used for the radar measurement is shown in Fig. 4, where β represents elevation angle; θ is the azimuth angle; and the X_I - Y_I - Z_I frame is inertial.

3. Rendezvous Navigation Computation.

In this portion of the system, the position and velocity of the LEM in inertial space are estimated along with the measurement biases. Basically, this is accomplished by tracking the CM and utilizing this tracking data, at discrete time intervals, along with a priori statistical knowledge (LEM position and velocity deviations from a reference trajectory, measurement random errors and measurement biases) to obtain an optimum linear estimate. It is inherently assumed, that the ephemeris of the CM is precisely known in inertial space so that determining the LEM's position and velocity, with respect to the CM, determines the LEM's inertial position and velocity. The fact that the CM ephemeris is not exactly known in no way affects the determination of the LEM's relative position and velocity which is of first order importance in the rendezvous problem. The estimate of the LEM's inertial position and velocity will be in error, but this is a second order effect, with negligible influence on mid-course and rendezvous guidance.

The details of the navigation scheme may be more readily explained with the aid of the block diagram in Fig. 5. The concept of a reference trajectory is utilized to permit the use of perturbation theory, i. e., estimates are made of position and velocity deviations from a reference trajectory. To further assure the validity of the perturbation theory, the reference trajectory used is that of the current estimated trajectory, so that deviations from this reference are always small.

Measurements are utilized and the estimates updated at discrete times (typically every 60 seconds during the mid-course phase), thus allowing time for some preliminary measurement smoothing and navigation computation time. The following initial inputs are required after which, at the specified time intervals, the LEM's position and velocity estimates are updated, as are the measurement bias estimates:

Required Initial and Tracking Inputs

A. Statistical Initial Inputs.

1. covariance matrix of LEM initial position and velocity errors $(E_{LEM})_0$ - six by six.

2. covariance matrix of initial bias estimation errors $(E_{BIAS})_0^{\dagger}$ - three by three.

3. variance of tracking measurement errors. (for each type of measurement used).

† Since there is no correlation between initial deviations and biases, these correlation terms in the nine by nine initial covariance matrix (E_{LEM}) - nine by nine - are set equal to zero.

B. Reference Trajectory Inputs.

1. LEM inertial position and velocity vectors $(\underline{R}_{LEM})_0$, $(\underline{V}_{LEM})_0$.

2. CM inertial position and velocity vectors $(\underline{R}_{CM})_0$, $(\underline{V}_{CM})_0$.

3. aim point vector $\underline{R}_{CM}(T_A)$.

4. nominal arrival time T_A.

5. velocity correction criteria ratio (described in section 4).

C. Tracking Measurements.

1. type of measurements to be used: Range (R), Range Rate ($\dot{R}$), Elevation Angle (β), Elevation Angle Rate ($\dot{\beta}$), Azimuth Angle (θ), or Azimuth Angle Rate ($\dot{\theta}$).†

2. time interval to be employed for estimate update.

D. Initial Estimates.

1. position and velocity deviations = 0.
2. bias = 0.

With reference to Fig. 5, the estimation procedure at the first time point, t_1 (e.g., 60 seconds from burnout injection) may be traced through the diagram starting at the initial reference trajectory parameters. The equations of motion (Fig. 6) are integrated to yield LEM and CM position and velocities at t_1 ($\hat{\underline{R}}'_{LEM}$, $\hat{\underline{V}}'_{LEM}$, $\underline{R}'_{CM}$, $\underline{V}'_{CM}$). $\hat{\underline{R}}'_{LEM}$ is required in the statistical computation section of the system as explained in the following section. Since the reference trajectory is defined as the current estimated LEM trajectory, "hats" appear over $\hat{\underline{R}}'_{LEM}$ and $\hat{\underline{V}}'_{LEM}$ to indicate estimates. Subtracting the LEM parameters from the CM parameters yields

† Any combination of these measurements may be employed, but as presently configured, the system can estimate biases in only 3 measurements.

the current estimate of the relative trajectory parameters ($\hat{\underline{R}}'_{CL}$, $\hat{\underline{V}}'_{CL}$). These relative parameters are used for two computations: one, the measurement geometry vector (b-vector); and two, the estimate of the measurement to be made ($\hat{q}'$). Each type of measurement has its appropriate b-vector, which is the quantity relating the deviation in the measurement to the deviation in the state vector. Typical b-vectors are given in Fig. 6 for the set of three measurements ($\dot{R}$, β, θ) normally used. (b-vectors for all six radar measurements, $R, \dot{R}$, β, θ, $\dot{\beta}$, $\dot{\theta}$ may be found in Ref. 3.) The estimate of the value of the measurement to be made is computed using the appropriate equation in Fig. 7 for $\dot{R}_{CL}$, β or θ. (It should be noted here that when more than one type of measurement is utilized, each measurement is processed independently. Although the measurements are made simultaneously at time t_1, they are utilized sequentially in the computations to update the estimate of the LEM position and velocity.)

With reference to Fig. 5, the b-vector is used in two computations: first, the weighting vector ($\underline{W}$); and second, statistical computation (S-C) section of the system. $\underline{W}$ is computed as shown in Fig. 5 using $\underline{b}$, the extrapolated covariance matrix (E'_{LEM}) - nine by nine - which comes from the statistical computation section, and the variance of the random measurement error ($\bar{\alpha}^2$). Then $\underline{W}$ is: one, fed back into the S-C section to be used in updating E'_{LEM} -nine by nine- for the next time point; and two, used to compute the optimum estimate of the state deviation vector ($\delta\hat{\underline{x}}$).

The optimum linear estimator requires four quantities at time t_1: first, the weighting vector ($\underline{W}$); second, the estimate of the measured quantity ($\hat{q}'$); third, the actual measurement (q); and fourth, the current bias estimate ($\hat{\underline{B}}$IAS). Initially, the bias estimate is zero, but after t_1, there will exist a value for this parameter which has been extrapolated from the last time point. With these quantities, the current estimate of the state deviation vector ($\delta\hat{\underline{x}}$) is computed. (i. e., the position and velocity deviation from the current estimated position and velocity plus the bias estimate in each of the measurements). It should be noted by observing the $\underline{b}_6$ vector for each measurement (Fig. 6), that only the bias estimate pertaining to the measurement being made is up dated, even though all these bias estimates ($\underline{B}IAS$) are included in the equations.

Once $\delta\hat{\underline{x}}$ has been computed at t_1, the new reference trajectory is formed by adding $\delta\hat{\underline{r}}$ to $\hat{\underline{R}}'_{LEM}$ and $\delta\hat{\underline{v}}$ to $\hat{\underline{V}}'_{LEM}$. These new parameters are fed to the velocity correction section of the system. The bias estimate portion of $\delta\hat{\underline{x}}$, ($\hat{\underline{B}}IAS$) is stored until needed at the next time point. If the velocity correction logic has called for a velocity correction, the value of the correction applied ($\Delta\underline{V}$) in terms of IMU accelerometer output is used to further update the new reference trajectory. (NOTE: In Fig. 5, $\Delta\underline{V}$ is shown to be added impulsively to $\delta\hat{\underline{v}}$ for convenience.)

The entire procedure discussed above yields the best current estimate at time t_1 of the following parameters: one, the LEM's position and velocity; and second, the measurement biases. This procedure is repeated at each of the predetermined time intervals through the rendezvous phase. A slight modification is made at the start of terminal rendezvous maneuver. The bias estimate at that time is fixed, and no further bias estimates are made. This is done to reduce some of the computations and does not affect accuracy since a satisfactory estimate of bias has been achieved before the terminal rendezvous phase.

4. Rendezvous Statistical Computation.

The section of the system shown in block diagram form in Fig. 8 has three major functions: first, computation of the transition matrix (Φ) - six by six-; two, extrapolation of the matrices E_{LEM}- nine by nine- and X-six by six-; and three, updating E_{LEM}- nine by nine-after a measurement and updating X-six by six- after a velocity correction. The E_{LEM} - six by six-matrix may also be updated after a velocity correction, if a substantial error is expected in applying a velocity correction. For the expected errors in application, this has been found to be unnecessary. The equations required for the extrapolation and updating functions are listed in Fig. 9. The differential equation which is integrated for the computation of Φ - six by six - is given in Fig. 7, together with the initial condition for Φ - six by six. The explicit expression for the three by three G matrix (gradient of gravity with respect to position) is:

$$G = \frac{\mu}{R^5} \begin{bmatrix} 3R_x^2 - R^2 & 3R_xR_y & 3R_xR_z \\ 3R_yR_x & 3R_y^2 - R^2 & 3R_yR_z \\ 3R_zR_x & 3R_zR_y & 3R_z^2 - R^2 \end{bmatrix}$$

where

R = magnitude of $\hat{\underline{R}}_{LEM}$

μ = gravitational constant

R_x, R_y, R_z = components of $\hat{\underline{R}}_{LEM}$

The initial E_{LEM} - six by six - and E_{BIAS} - three by three - are initial input data (as explained previously) which are combined to give $(E_{LEM})_0$ - nine by nine. This initial nine by nine matrix is extrapolated to yield the required E'_{LEM} - nine by nine - to be used in computing $\underline{W}$ in the navigation section. Then, together with $\underline{b}$ and $\underline{W}$ (from the navigation section), E'_{LEM} - nine by nine - is used to update itself, yielding E_{LEM} - nine by nine - at time t_1. E_{LEM} - nine by nine - is then fed back in Fig. 8 and extrapolated to the next point for the subsequent estimate update. The six by six portion of E_{LEM} - nine by nine - is sent to the velocity correction section to be used in the statistical correction logic. The X - six by six - matrix is also required in the statistical correction logic, and is the covariance matrix of true deviations. Since the initial deviation estimate is zero, the error in the estimate is just the true deviation. Thus, the initial value of X - six by six - is $(E_{LEM})_0$ six by six - as indicated in Fig. 8. If a velocity correction is made, the extrapolated value of the X matrix at that time must be updated since the true velocity deviation has been changed. (This is assumed to be an impulsive velocity correction and introduces very little error.)

5. Velocity Correction Computation and Decision.

This section of the system is subdivided into the mid-course velocity correction, and terminal rendezvous velocity correction since a modification to the logic is made when the terminal rendezvous phase is initiated.

A. Midcourse velocity correction.

Two separate logic schemes have been considered for determining when a velocity correction should be applied during the mid-course rendezvous phase. One would be simply to have predetermined times along the trajectory at which the estimated correction ($\Delta\hat{\underline{V}}$) would be applied. In such a system, the final correction could always be applied at some predetermined range (e.g., 25 n.m.) which would limit the miss distance at the nominal arrival time to a reasonable value. If the trajectories to be flown were fairly well establihsed, this scheme allows for a degree of optimization by properly selecting the correction times to minimize the total ΔV. However, in order to have a logic which is satisfactory for a wide variety of trajectories, though not necessarily optimum for any one, a statistical velocity correction (SVC) logic has been incorporated.

The SVC logic utilizes a priori statistical knowledge of the LEM's position and velocity deviations (X - six by six - matrix) and the updated statistical knowledge of errors in the estimates of these deviations (E_{LEM}) - six by six - to determine the mean squared estimate of the required velocity correction (DELV), and the mean squared uncertainty in this estimate (DELU). When the square root of the ratio of DELU to DELV is below a predetermined level, (RATIO), the estimated velocity correction ($\Delta\hat{\underline{V}}$) is applied. Figure 10 illustrates this system. Utilizing only the initial reference trajectory $(\underline{R}_{LEM})_0$, $(\underline{V}_{LEM})_0$ and the time of arrival (T_A) for which the velocity correction will achieve an intercept trajectory, the C^* matrix is calculated at each measurement time point along the trajectory. The C^* matrix contains partial derivatives of required velocity for an intercept at T_A with respect to position deviations at the present time. General equations for C^* are given in Fig. 11, and more specifically, in Ref. 1 under perturbation matrices. DELV and DELU are then calculated using equations in Fig. 11. A small degree of error is introduced since only the original reference trajectory is used, wheras E_{LEM} and X' are propagated along the estimated trajectory. Since the deviations between these trajectories is always quite small, however, this error produces negligible affect on the values of DELV and DELU.

The estimate of the velocity correction required ($\Delta\hat{\underline{V}}$) is made on the basis of a constant arrival time (T_A). This computation is shown in Fig. 10. The position vector of the command module ($\underline{R}_{CM}(T_A)$) at time = T_A is available as initial data. This vector, together with the current estimate of the LEM position vector ($\hat{\underline{R}}_{LEM}$) and the time desired for an intercept (the difference between the initial desired arrival time (T_A) and the present time (TIME)) are fed into a computational scheme for solving Lambert's problem. The velocity required by the LEM to intercept the CM at t = T_A is computed as ($\underline{V}_C$). By subtracting the current estimate of the LEM's velocity ($\hat{\underline{V}}_{LEM}$) from $\underline{V}_C$, $\Delta\hat{\underline{V}}$ is obtained. When the velocity correction logic demands application, $\Delta\hat{\underline{V}}$ is commanded and the output of the IMU yields the actually applied $\Delta\underline{V}$ which is returned to the navigation computation section to update the LEM's estimated trajectory.

Mention should be made of the affect of errors in the knowledge of the aim point - $\underline{R}_{CM}(t_A)$, caused by uncertainties in the CM ephemeris. In a rendezvous problem where the vehicle being tracked is also the target, aim point errors are small second order effects. It was mentioned previously that the navigation system accurately defines the relative position and velocity of the LEM with respect to the CM, although the estimates of inertial position and velocity may be in error due to CM ephemeris uncertainties. Thus, the inertial estimate is degraded in order to place the LEM in a correct relative position to the CM. Then, for the relatively short flight time trajectories involved, the estimated $\Delta\hat{\underline{V}}$ required to intercept using incorrect inertial data for <u>both</u> vehicles is negligibly different from that required using true inertial data.

A special situation must be accounted for during the mid-course velocity correction phase. This is when a velocity correction is called for and the central angle from $\hat{\underline{R}}_{LEM}$ and the aim point vector $\underline{R}_{CM}(T_A)$ is in the vicinity of 180 degrees. If the trajectory is not absolutely coplanar (which is unrealistic), the velocity correction computed by solving Lambert's problem is prohibitively high. Logic must be provided, therefore, to prevent application of the correction until the central angle becomes smaller than 180 degrees. For the trajectories studied, which were normally noncoplanar due to launch conditions and system errors, preventing mid-course velocity corrections in a band $\pm$ 20 degrees about 180 degrees proved satisfactory.

B. Terminal rendezvous velocity correction.

A slight modification to the basic guidance and navigation scheme discussed for the mid-course phase is made in the terminal rendezvous phase. This amounts simply to a redefinition of the aim point and the desired time of arrival (Fig. 12).

The objective of the terminal rendezvous phase is to control the relative closing velocity to zero as the range between the two vehicles closes to a desired terminal separation range from which docking can be achieved. Since the mid-course rendezvous phase established an intercept trajectory between the two vehicles, the relative velocity can be considered to be range rate as measured by the rendezvous radar. Under these conditions, the terminal rendezvous maneuver can be described in a range-range rate (R-$\dot{R}$) phase plane by some criteria which controls the closing velocity ($\dot{R}$) as some function of range (R) so that the desired terminal conditions can be established. There are many terminal R-$\dot{R}$ criteria or schedules that could be used. These generally fall into categories such as parabolic, linear or a fixed range-range rate schedule. The guidance scheme shown in Fig. 12 is general in the sense that it could be used with any terminal R-$\dot{R}$ criteria provided tracking measurements (at least one), could be made between thrust periods.

The type of terminal R-$\dot{R}$ criteria used in the primary G&N system will depend upon the following factors:

1) The maximum closing relative velocities expected for rendezvous trajectories initiated from noncoplanar launch conditions, or direct abort trajectories from any point after separation.

2) The propulsion system or systems that must be used to effect the terminal rendezvous maneuver for either the LEM or CSM.

3) Monitoring requirements (visual and system displays), of the astronauts from both LEM and CSM and the degree of desired similarity or compatibility.

4) Back-up guidance requirements (possibly manually controlled in the CSM and visually and/or manually controlled in the LEM).

At the present time, an "accepted" terminal R-$\dot{R}$ criteria covering all of these factors has not been established. Some typical criteria that have been used in the analysis of the primary G&N system for both LEM and CSM controlled rendezvous are as follows:

<u>LEM controlled terminal rendezvous</u>

<u>Parabolic R-$\dot{R}$ criteria starting at R = 5 n. m.</u>

Engine on: $\dot{R}^2/2R \geq 1/3 \text{ fps}^2$

Engine off: $\dot{R}^2/2R \leq 1/6 \text{ fps}^2$

<u>Fixed Range-Range rate schedule</u>

Range	Desired range rate
5 n. m.	-100 fps
1. 5 n. m.	-20 fps
. 25 n. m.	-5 fps

These velocity corrections are nominally controlled by the LEM RCS jets except in those abort or ascents from large out of plane conditions where the descent or ascent engines are required for the first correction to establish closing conditions within the RCS capability.

CSM controlled terminal rendezvous

Fixed Range-Range rate schedule

Range	Desired range rate
5 n. m.	-80 fps
0. 5 n. m.	-5 fps

These two terminal velocity corrections are made with the SM propulsion system. The SM RCS has been assumed capable of correcting the terminal closing velocity of -5 fps to within 1 fps.

These criteria result in 3 to 6 thrust periods for the LEM controlled rendezvous, while the CSM rendezvous is restricted to 2 thrust maneuvers in order to limit SM engine restarts. The time required for the terminal rendezvous maneuver using the above LEM R-$\dot{R}$ criteria range from 7 to 10 minutes over the last 5 n. m. while the CM criteria results in a terminal phase of 5-6 minutes. The desired docking conditions at the end of the guidance controlled terminal rendezvous maneuver have been a separation range of 500 feet with closing velocity of -5 fps $\pm$ 1 fps. The primary point to be made here is that the primary G&N rendezvous technique in both vehicles is capable of performing virtually any terminal R-$\dot{R}$ criteria that may be specified.

As indicated in the diagram of Fig. 13 and equations of Fig. 12, the terminal R-$\dot{R}$ criteria programmed in the AGC is used to compute a new time to go (T_{GO}) to the intercept point. This intercept is defined by a new aim point along the CM orbit $\underline{R}_{CM}(T'_A)$ computed by integrating the CM equations of motion ahead by T_{GO} seconds from the present CM conditions ($\underline{R}'_{CM}$, $\underline{V}'_{CM}$). As indicated in Figures 12 and 13, the new time of arrival (T_A) is simply T_{GO} added to the present time. Then, as in the mid-course rendezvous velocity correction of

Fig. 10, the new arrival time (T'_A), the new aim point and current estimate of the LEM's position vector ($\hat{\underline{R}}_{LEM}$) are applied to a routine which solves Lambert's problem to yield the required LEM velocity ($\underline{V}_c$) which will result in an intercept at T'_A. The $\Delta\hat{\underline{V}}$ is again $\underline{V}_c$ minus the current estimate of the LEM's velocity ($\hat{\underline{V}}_{LEM}$). This entire procedure is illustrated in Fig. 13. It is apparent that this scheme, besides taking out the required $\dot{R}$, also makes appropriate corrections normal to the line of sight to maintain the vehicles on a collision course.

It should be noted that the manner in which T_{GO} is calculated ($R/\dot{R}_d$) assumes an impulsive thrust. This follows since the solution to Lambert's problem requires an impulsive velocity correction. Since the thrust is applied in a finite time, the actual time to rendezvous will be shorter than the computed T_{GO}. However, T_{GO} is redefined at subsequent velocity correction points until the final $\dot{R}$ cancellation requires a small velocity application resulting in a small error in T_{GO} due to finite thrust times. In addition, the closing velocity will also be small at this time, making small errors in T_{GO} negligible.

The navigation scheme used during the mid-course phase continues right into the terminal rendezvous phase computing $\hat{\underline{R}}_{LEM}$ and $\hat{\underline{V}}_{LEM}$. One slight modification mentioned previously is the fixing of the last estimate of the measurement biases at the start of this phase. Another modification which may be initiated in the terminal phase(or quite possibly earlier in the mid-course phase), is to increase the magnitude of the covariance matrix E_{LEM}† - six by six. This has the effect of increasing the sensitivity of the estimation process by effectively increasing the "gain" of the system. (A similar procedure is followed in the translunar mid-course navigation phase.) The theory

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† If the decision to increase the magnitude of E_{LEM} - six by six- is made, the bias estimate is fixed at that point and no longer estimated, (i. e., E_{LEM} - nine by nine- becomes E_{LEM}-six by six). This is necessary since changing only E_{LEM} - six by six-would make E_{LEM}- nine by nine non-positive definite. Nothing is lost by dropping the bias estimate at this point since this estimate will be sufficiently accurate by that time.

behind this is that after many measurements have been taken, the estimation errors (in a statistical sense), will have become very small, i.e., E_{LEM} becomes very small. The effect of this on the estimation process is to place little weight on any additionally received measurements, and rely heavily on the current estimates. Thus, by increasing the magnitude of E_{LEM}, the measurements currently received, which happen to be extremely good because of the small range, are given added weight and may substantially enhance the accuracy of the estimates. The manner in which this modification is presently being employed is as follows: whenever the mean square position errors (the trace of the upper left hand three by three submatrix of E_{LEM}) becomes less than $(300 \text{ feet})^2$, every term of E_{LEM} is multiplied by ten. The results to date have been quite satisfactory.

It should be pointed out that the diagrams of Figures 10 and 13 are computational flow diagrams and do not represent detailed schematics of the interface between the primary G&N system and the spacecraft SCS and propulsion systems. The $\underline{\Delta V}$ signals in these figures are commanded vector velocity corrections. In the spacecraft, this commanded velocity correction would be presented to SCS and thepropulsion systems as an attitude command, an engine-on signal, followed by an engine-off signal after the desired velocity correction had been achieved as measured by the IMU. The engine-on signal in Fig. 10, mearly represents the output of the velocity correction criteria and is not necessarily the same engine-on signal from the AGC to the spacecraft flight control system.

LIST OF REFERENCES

1. Battin, R. H., "A Statistical Optimizing Navigation Procedure for Space Flight," MIT Report R-341, Cambridge, Massachusetts, Revised May 1962.

2. "Radar Requirements for Primary Guidance and Navigation Operation," MIT Report R-404, Cambridge, Massachusetts, April 1963, Confidential.

3. Muller, E. S., "Summary of b-Vectors for Radar Measurements," SGA Memo #43, Cambridge, Massachusetts, May 7, 1963.

RENDEZVOUS GUIDANCE FLOW DIAGRAM

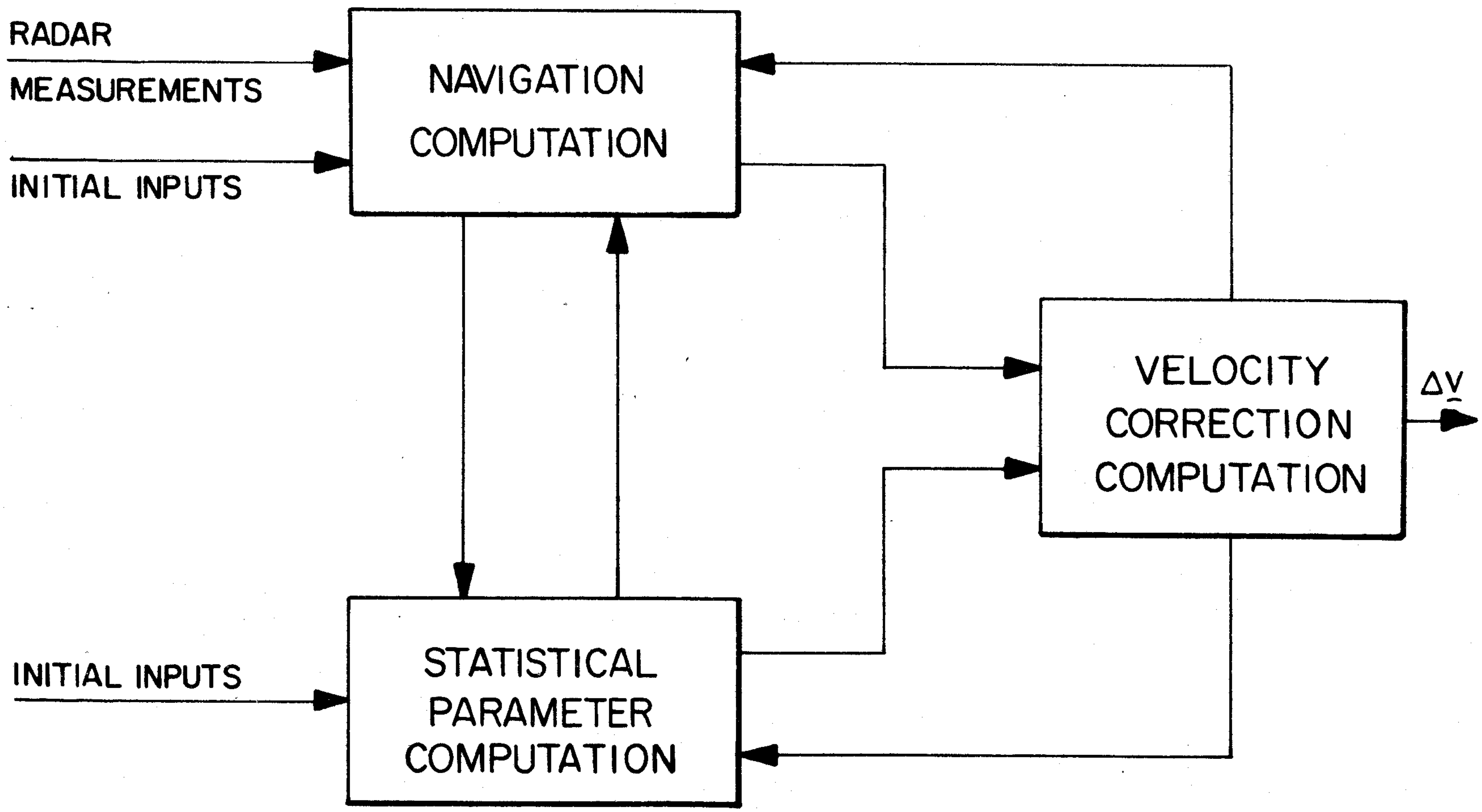

Figure 1

GUIDANCE EQUATION NOTATION

1. GENERAL

A = SCALAR

$\underline{A}$ = n-DIMENSIONAL VECTOR

$\hat{\underline{A}}$ = ESTIMATED VECTOR

$\underline{A}'$ = EXTRAPOLATED VECTOR FROM LAST MEASUREMENT

$\underset{(n \times m)}{A}$ = MATRIX (n BY m)

$\underset{(n \times m)}{A^T}$ = TRANSPOSE OF MATRIX

$\bar{A}$ = STATISTICAL AVERAGE

$[A]_0$ = INITIAL CONDITION

2. SUB-SCRIPTS

LEM : LUNAR EXCUSION MODULE

CM : COMMAND MODULE

CL : RELATIVE CM-LEM QUANTITY

Figure 2

GENERAL DEFINITIONS

$$\delta \underline{X} = \begin{bmatrix} \delta \underline{r} \\ \delta \underline{v} \\ \underline{\text{Bias}} \end{bmatrix} \quad : \text{(STATE DEVIATION VECTOR)}$$

$$\underline{e} = \delta \hat{\underline{X}} - \delta \underline{X} = \begin{bmatrix} \underline{\epsilon} \text{ (POS.)} \\ \underline{\delta} \text{ (VEL.)} \\ \underline{\gamma} \text{ (BIAS)} \end{bmatrix} \quad : \text{(ERROR IN ESTIMATED } \delta \underline{X}\text{)}$$

$$\underset{(9\text{X}9)}{E_{LEM}} = \overline{\underline{e}\,\underline{e}^T} = \left[\begin{array}{cc|c} \overline{\underline{\epsilon}\underline{\epsilon}^T} & \overline{\underline{\epsilon}\underline{\delta}^T} & \overline{\underline{\epsilon}\underline{\gamma}^T} \\ \overline{\underline{\delta}\underline{\epsilon}^T} & \overline{\underline{\delta}\underline{\delta}^T} & \overline{\underline{\delta}\underline{\gamma}^T} \\ \hline \overline{\underline{\gamma}\underline{\epsilon}^T} & \overline{\underline{\gamma}\underline{\delta}^T} & \overline{\underline{\gamma}\underline{\gamma}^T} \end{array}\right] = \left[\begin{array}{c|c} \underset{(6\text{X}6)}{E_{LEM}} & ---- \\ \hline \vdots & \underset{(3\text{X}3)}{E_{Bias}} \end{array}\right] \quad : \text{COVARIANCE MATRIX OF ESTIMATION ERRORS}$$

$$\underset{(6\text{X}6)}{X} = \overline{\begin{bmatrix} \delta \underline{r} \\ \delta \underline{v} \end{bmatrix} \begin{bmatrix} \delta \underline{r} \\ \delta \underline{v} \end{bmatrix}^T} \quad : \text{(COVARIANCE MATRIX OF DEVIATIONS)}$$

GENERAL EXTRAPOLATION: $\delta X' = \underset{(9\text{X}9)}{P}\, \delta X$; $\begin{bmatrix} \delta \underline{r} \\ \delta \underline{v} \end{bmatrix}' = \underset{(6\text{X}6)}{\Phi} \begin{bmatrix} \delta \underline{r} \\ \delta \underline{v} \end{bmatrix}$

$$\underset{(9\text{X}9)}{P} = \left[\begin{array}{cc|c} \multicolumn{2}{c|}{\underset{(6\text{X}6)}{\Phi}} & \begin{array}{c} 0 \\ \hline 0 \end{array} \\ \hline 0 & 0 & \underset{(3\text{X}3)}{\Phi_{Bias}} \end{array}\right]$$

Figure 3

MEASUREMENT COORDINATE SYSTEM

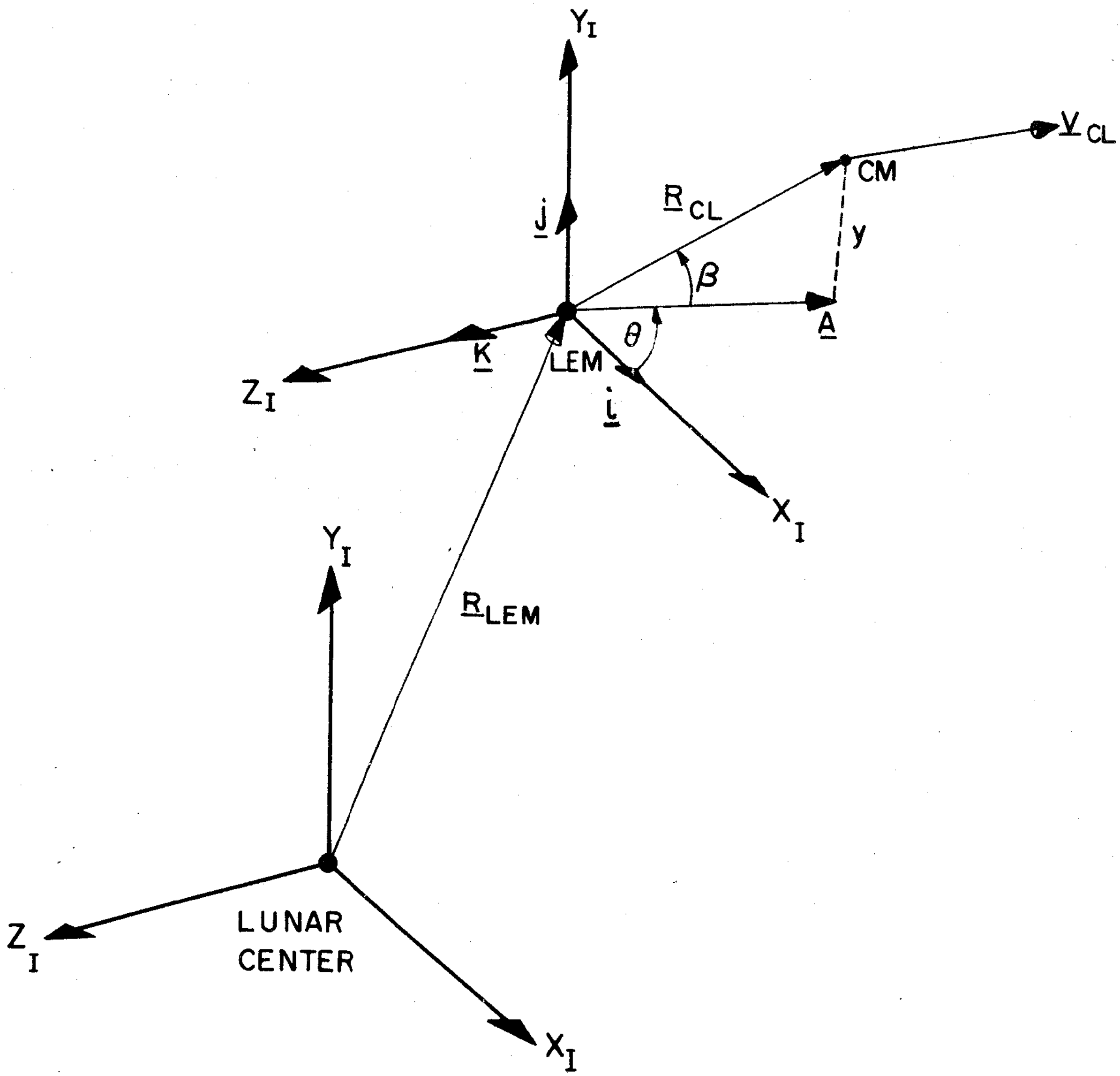

Figure 4

RENDEZVOUS NAVIGATION COMPUTATION

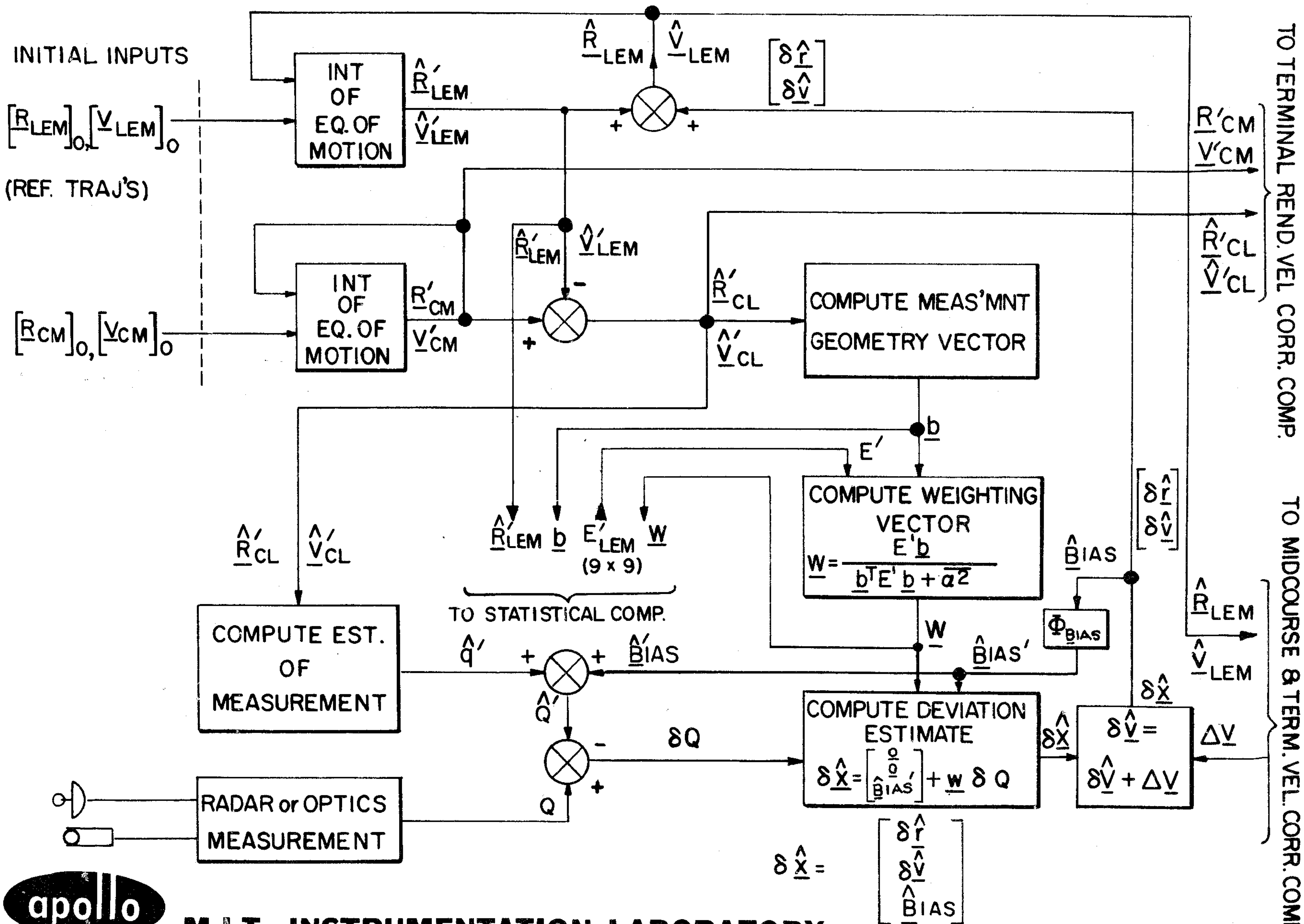

Figure 5

Figure 6

NAVIGATION GUIDANCE EQUATIONS

INTEGRATION OF EQUATIONS OF MOTION:

$$\ddot{\underline{R}} = \dot{\underline{V}} = -\frac{\mu}{R^3}\underline{R}$$

MEASUREMENT GEOMETRY VECTOR $\underline{b}$:

DEFINED AS: $\delta q = \underline{b} \cdot \delta \underline{X}$ = DEVIATION OF MEASUREMENT FROM REF.

$$\underline{b} = \begin{bmatrix} \underline{b}_0 \\ \underline{b}_3 \\ \underline{b}_6 \end{bmatrix}$$

MEASUREMENT:	$\underline{b}_0$	$\underline{b}_3$	$\underline{b}_6$
RANGE RATE	$\frac{1}{R_{CL}^3}\left[\underline{R}_{CL} \times (\underline{V}_{CL} \times \underline{R}_{CL})\right]$	$\underline{R}_{CL}/R_{CL}$	$(1,0,0)$
ANGLE (β)	$\frac{1}{R_{CL}^2 Ay}\left[A^2\underline{R}_{CL} - R_{CL}^2\underline{A}\right]$	$(0,0,0)$	$(0,1,0)$
ANGLE (θ)	$\frac{1}{A^2 y}\left[\underline{R}_{CL} \times \underline{A}\right]$	$(0,0,0)$	$(0,0,1)$

$\overline{\alpha^2}$ = RADAR OR OPTICAL TRACKING VARIANCE

NAVIGATION GUIDANCE EQUATIONS (CONT.)

EXTRAPOLATED ESTIMATE OF MEASUREMENTS:

$$q = \dot{R}_{CL}, \beta, \theta$$

$$\dot{R}_{CL} = \frac{\underline{R}_{CL}}{R_{CL}} \cdot \underline{V}_{CL}$$

$$\beta = \sin^{-1}\left(\frac{\underline{R}_{CL}}{R_{CL}} \cdot \underline{j}\right)$$

$$\theta = \sin^{-1}\left(-\underline{R}_{CL} \cdot \underline{k}/A\right)$$

Figure 7

TRANSITION MATRIX P:

$$\underset{(6\times 6)}{\dot{\Phi}} = \underset{(6\times 6)}{F}\ \underset{(6\times 6)}{\Phi} \quad ; \quad \begin{bmatrix} \Phi \\ 6\times 6 \end{bmatrix}_0 = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

$$\underset{(6\times 6)}{F} = \begin{bmatrix} 0 & I \\ G & 0 \end{bmatrix} \quad ; \quad \underset{(3\times 3)}{G} = \nabla \underline{g} \quad ; \quad \nabla = \left\| \frac{\partial}{\partial \underline{R}} \right\|$$

$$\underset{(3\times 3)}{\Phi_{Bias}} = I$$

RENDEZVOUS STATISTICAL COMPUTATION

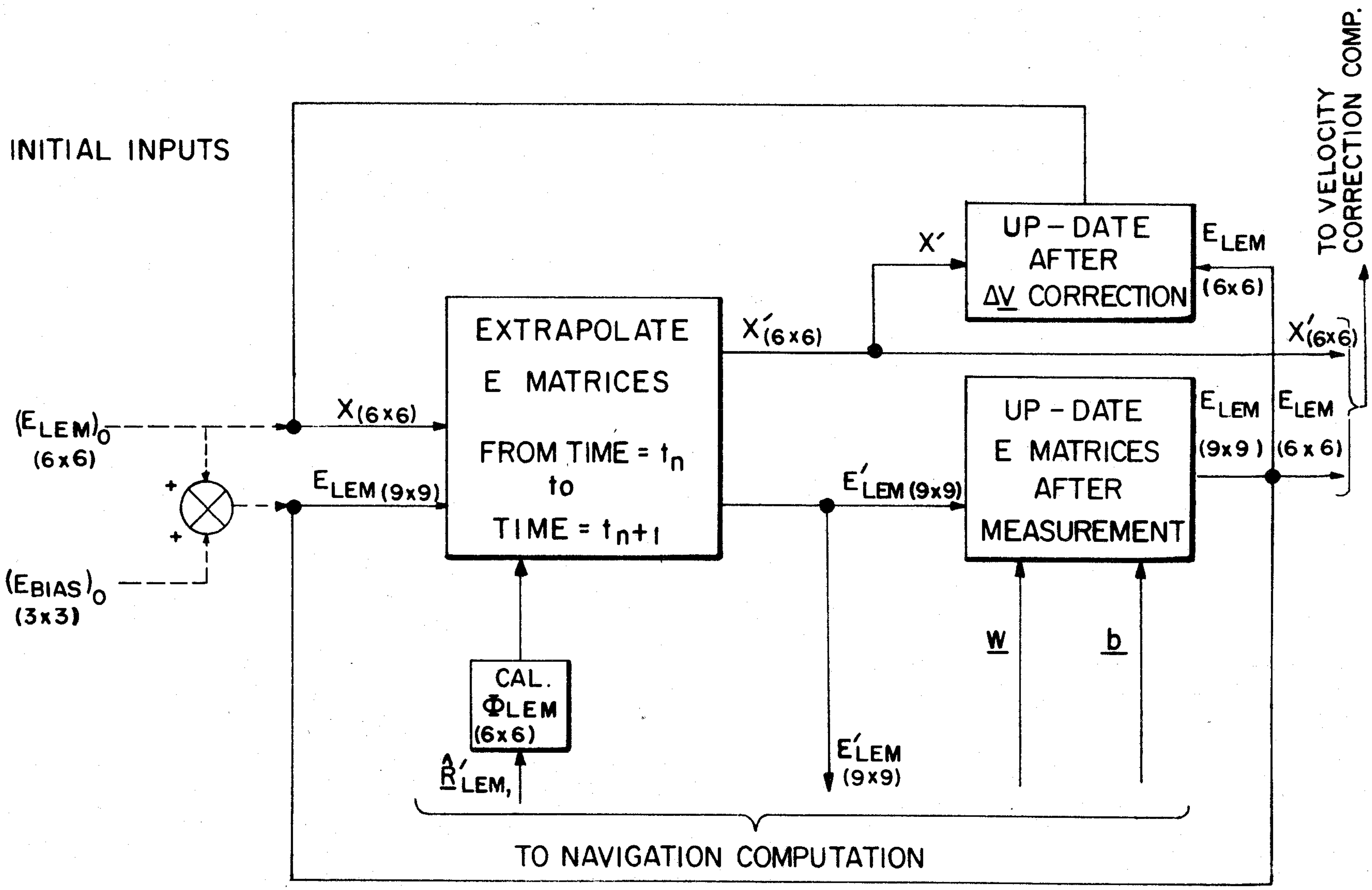

Figure 8

STATISTICAL EQUATIONS

EXTRAPOLATION OF E MATRICES

$$\underset{(9\times9)}{E'_{LEM}} = \underset{(9\times9)}{P} \; \underset{(9\times9)}{E_{LEM}} \; \underset{(9\times9)}{P^T}$$

$$\underset{(6\times6)}{X'} = \underset{(6\times6)}{\Phi_{LEM}} \; \underset{(6\times6)}{X} \; \underset{(6\times6)}{\Phi^T_{LEM}}$$

Figure 9

UP-DATING E MATRICES (9X9):

$$E = \left(I - \underline{w} \; \underline{b}^T\right) E'$$

UP-DATING OF X MATRIX (6X6):

$$X = \left(I + JB\right)\left(X' - \underset{(6\times6)}{E'_{LEM}}\right)\left(I + JB\right) + \underset{(6\times6)}{E'_{LEM}} ; \quad \underset{(6\times3)}{J} = \begin{bmatrix} 0 \\ \hline \underset{(3\times3)}{I} \end{bmatrix}$$

MID-COURSE RENDEZVOUS VELOCITY CORRECTION COMPUTATION AND DECISION

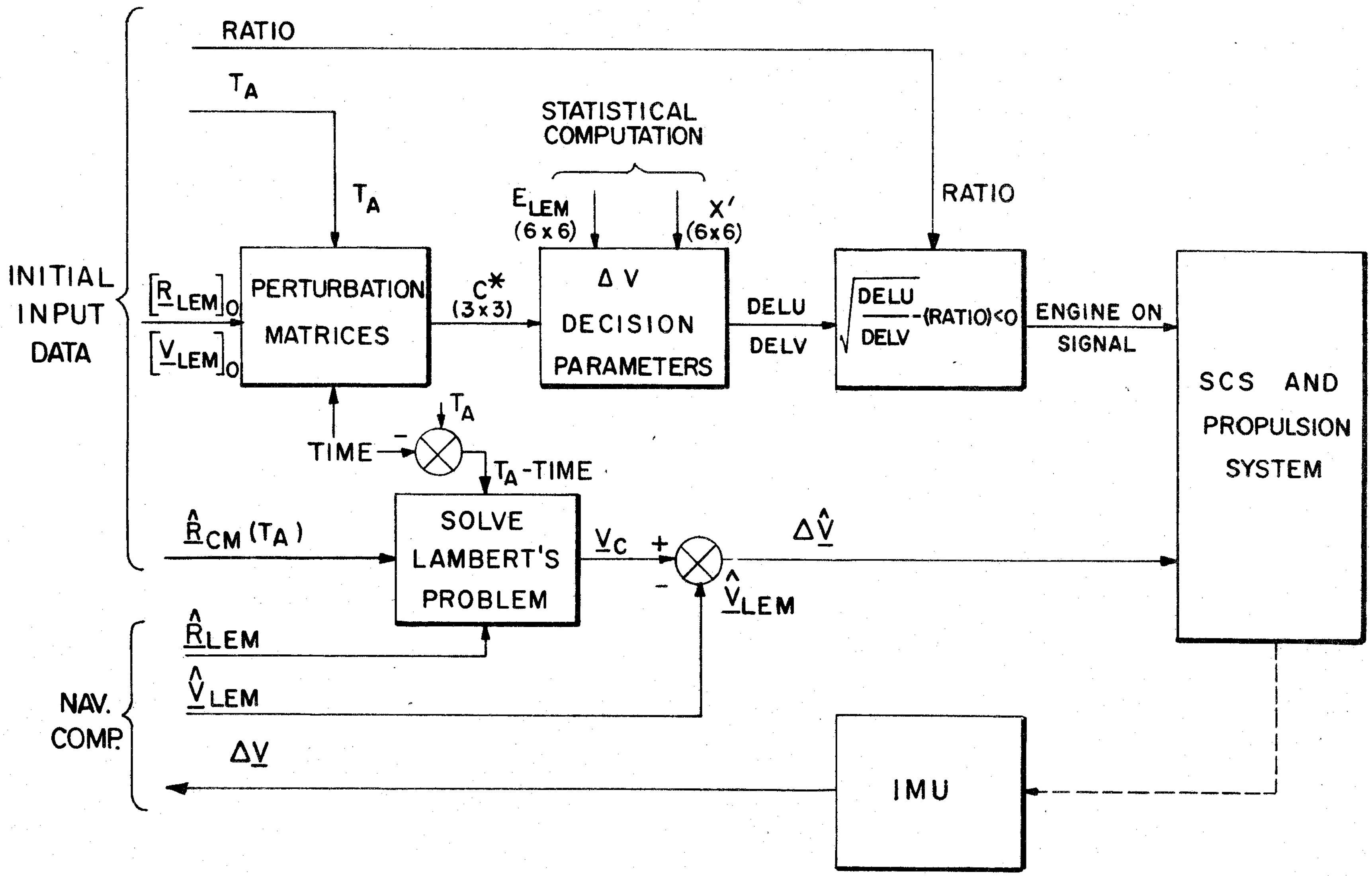

Figure 10

MID-COURSE RENDEZVOUS VELOCITY CORRECTION EQUATIONS

PERTURBATION MATRICES:

$$\underset{(3\times3)}{\nabla} = \left\| \frac{\partial}{\partial \underline{V}_{T_A}} \right\|$$

$$\underset{(3\times3)}{R^*} = \nabla \, \underline{R}_{LEM}$$

$$\underset{(3\times3)}{V^*} = \nabla \, \underline{V}_{LEM}$$

$$\underset{(3\times3)}{C^*} = \underset{(3\times3)}{V^*} \; \underset{(3\times3)}{R^{*-1}}$$

ΔV DECISION PARAMETERS:

$$\underset{(3\times6)}{B} = \left[\underset{(3\times3)}{C^*} \;\vdots\; \underset{(3\times3)}{-I} \right]$$

$$DELU = TRACE\left[\underset{(3\times6)}{B} \; \underset{(6\times6)}{E_{LEM}} \; \underset{(6\times3)}{B^T} \right]$$

$$DELV = TRACE\left[\underset{(3\times6)}{B} \; \underset{(6\times6)}{(X'} - \underset{(6\times6)}{E_{LEM})} \; \underset{(6\times3)}{B^T} \right]$$

Figure 11

TERMINAL RENDEZVOUS GUIDANCE

- MID-COURSE GUIDANCE WITH THE FOLLOWING MODIFICATIONS:

1. REDEFINE AIM POINT AND TIME OF ARRIVAL ON TERMINAL $R - \mathring{R}$ CONCEPT

$$T_{GO} = \frac{R \text{ (PRESENT RANGE)}}{\mathring{R}_d \text{ (DESIRED RANGE RATE)}}$$

$$T_A' = \text{TIME} + T_{GO}$$

INTEGRATE CM POSITION AHEAD BY T_{GO} FOR NEW AIM POINT

2. FIX ANGLE BIAS ESTIMATE AT PRESENT VALUE

Figure 12