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Space Guidance Analysis Memo #1-66

TO:	SGA Distribution
FROM:	T. C. Lu (NAA Technical Rep.)
DATE:	14 January 1966
SUBJECT:	Apollo Spacecraft-Engine Equations and Actuator Load

The purpose of this memo is to present the pitch-plane equations of motion of the combined spacecraft-engine system, and the actuator load versus the spacecraft pitch rate.

Summary

The combined spacecraft-engine system is essentially composed of two rigid bodies, Apollo spacecraft and SPS engine, joined at their pivot point (neglecting the structural flexibility and propellant slosh effects). A set of four equations containing five variables is obtained. The actuator load is computed.

Results

Using Fig. 1 and symbols defined in Table 1, we can write the equations of motion for the two rigid bodies, spacecraft and engine, as follows:

$$\begin{aligned} m_s \ddot{x}_s &= F_x \\ m_s \ddot{z}_s &= F_z \\ I_s \ddot{\theta} &= -K\delta + M + F_x \ell_x \sin\theta + F_z \ell_x \cos\theta \end{aligned} \tag{1}$$

and

$$\begin{aligned} m_e \ddot{x}_e &= -F_x + T\cos(\delta - \theta) \\ m_e \ddot{z}_e &= -F_z + T\sin(\delta - \theta) \\ I_e(\ddot{\delta} - \ddot{\theta}) &= -C(\dot{\delta} - \dot{\theta}) - K\delta + M + F_x \ell_e \sin(\delta - \theta) \\ &\quad - F_z \ell_e \cos(\delta - \theta) \end{aligned} \tag{2}$$

where

$$\begin{aligned} \ddot{x}_e &= \ddot{x}_s + \ddot{x}_{se} \\ \ddot{z}_e &= \ddot{z}_s + \ddot{z}_{se} \end{aligned} \tag{3}$$

and

$$\ddot{x}_{se} = \ell_e [\sin(\delta - \theta)\ddot{\delta} + \cos(\delta - \theta)\dot{\delta}^2] + [\ell_x \sin\theta - \ell_e \sin(\delta - \theta)]\ddot{\theta}$$
$$- 2\ell_e \cos(\delta - \theta)\dot{\delta}\dot{\theta} + [\ell_x \cos\theta + \ell_e \cos(\delta - \theta)]\dot{\theta}^2$$

$$\ddot{z}_{se} = \ell_e [-\cos(\delta - \theta)\ddot{\delta} + \sin(\delta - \theta)\dot{\delta}^2] + [\ell_x \cos\theta + \ell_e \cos(\delta - \theta)]\ddot{\theta}$$
$$- 2\ell_e \sin(\delta - \theta)\dot{\delta}\dot{\theta} - [\ell_x \sin\theta - \ell_e \sin(\delta - \theta)]\dot{\theta}^2 \quad (4)$$

Equations (1) and (2) consist of six equations containing seven variables; x_s, z_s, θ, δ, M, F_x and F_z. Eliminating F_x and F_z in Eqs. (1) and (2) results in four equations containing five variables, x_s, z_s, θ, δ and M.

Actuator Load

The actuator load for a constant gimbal angle of small value is found by using Eqs. (1) through (4) to be as follows:

$$M = a\dot{\theta}^2 + b\dot{\theta} + c \quad (5)$$

where

$$a = -\frac{(Q \quad - R \quad) P \ell_x \ell_e}{Q + R}\delta$$

$$b = -\frac{QC}{Q + R}$$

$$c = \left(K - \frac{RPT\ell_x}{Q+R}\right)\delta \quad (6)$$

$$P = \frac{m_s}{m_s + m_e}$$

$$Q = I_s + P\ell_x(\ell_x + \ell_e)$$

$$R = \ell_e + P\ell_e(\ell_x + \ell_e)$$

For the spacecraft data given in Report R-533, "A Block II TVC Digital Autopilot Compensation for CSM Spacecraft," the actuator load (M) versus the spacecraft pitch rate ($\dot{\theta}$) for constant gimbal angle (δ) of 0, ±0.1, and ±0.2 rad., is shown in Fig. 2.

Table 1 Glossary of Symbols

Symbols	Definitions
OXYZ	Right-handed orthogonal coordinate system
i, j, k	Right-handed orthogonal unit vectors
$O_1X_1Y_1Z_1$	Inertial axes
$O_2X_2Y_2Z_2$	Spacecraft body axes with O_2 at its mass center
$O_3X_3Y_3Z_3$	Engine body axes with O_3 at the pivot point
O_4	Engine mass center
ℓ_x	Distance between O_2 and O_3, ft
ℓ_e	Distance between O_3 and O_4, ft
θ	Spacecraft pitch angle, rad
δ	Engine gimbal angle, rad
m_s	Spacecraft mass, slug
m_e	Engine mass, slug
I_s	Spacecraft moment of inertia about its mass center, slug-ft^2
I_e	Engine moment of inertia about its mass center, slug-ft^2
$\underline{r}_s$	Spacecraft position vector, $\underline{O_1 O_2} = x_s i_1 + z_s k_1$, ft.
$\underline{r}_e$	Engine position vector, $\underline{O_1 O_4} = x_e i_1 + z_e k_1$, ft.
$\underline{r}_{se}$	$\underline{r}_e - \underline{r}_s$, $\underline{O_2 O_4} = x_{se} i_1 + z_{se} k_1$, ft.
$\underline{F}$	Force acted on spacecraft by hinge = $F_x i_1 + F_z k_1$, lb.
K	Hose compliance, ft-lb/rad.
C	"Jet" damping coefficient, ft-lb/rad/sec.
M	Actuator moment, ft-lb.
$\underline{T}$	Engine thrust, lb.

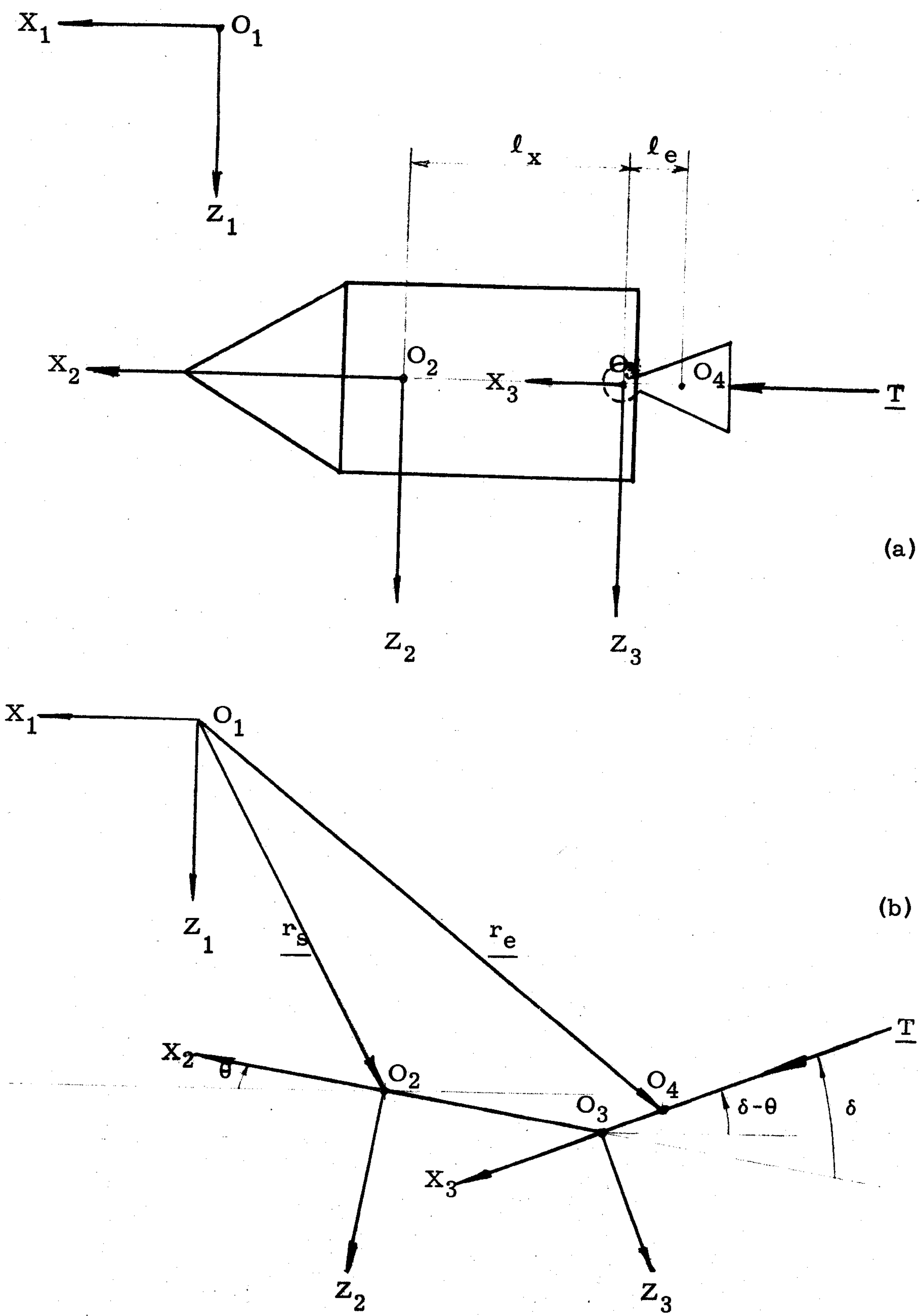

Figure 1
Apollo Spacecraft and SPS Engine System

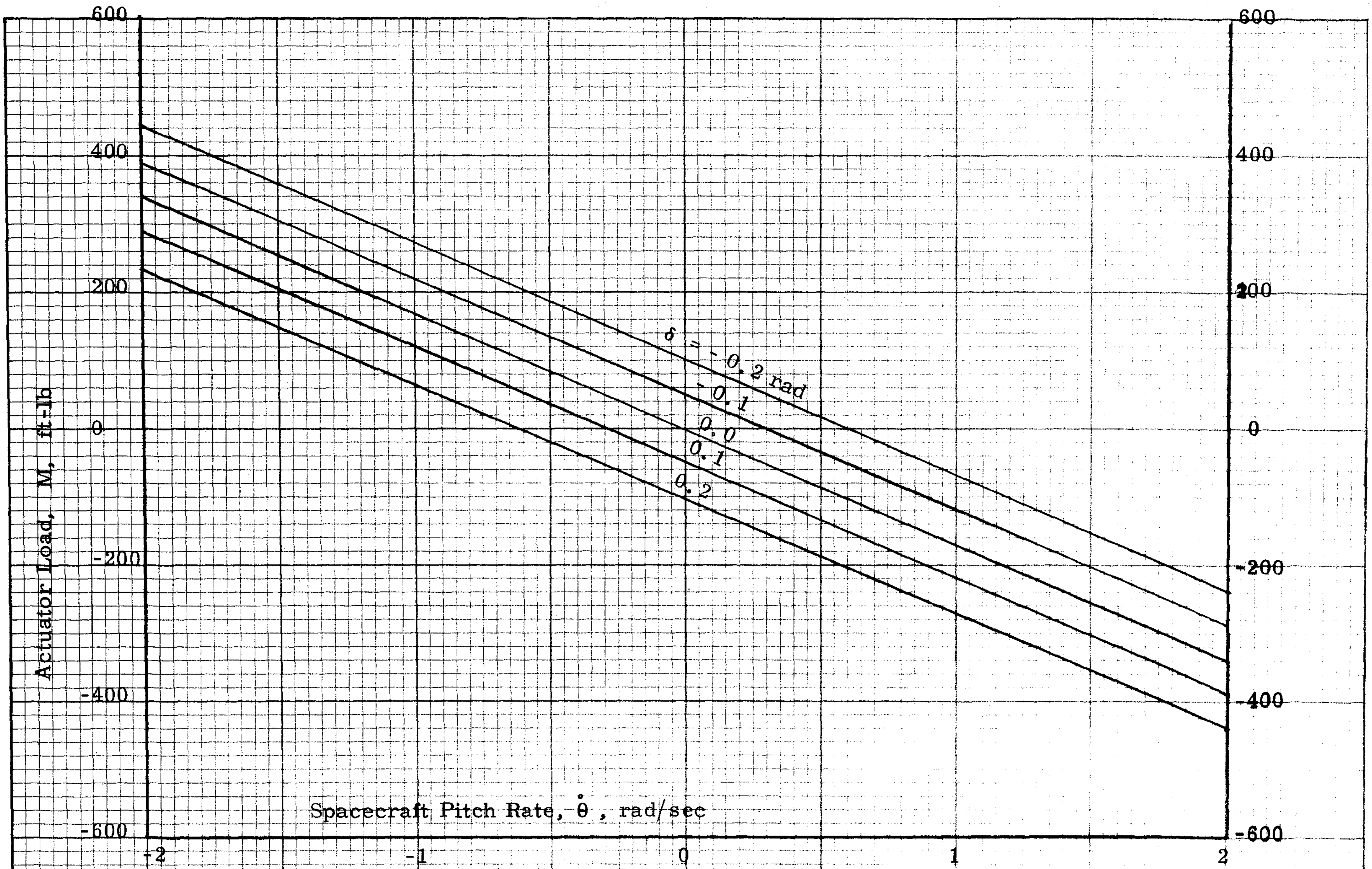

Figure 2 - Actuator Load Versus Spacecraft Pitch Rate for Constant Gimbal Angle