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Space Guidance Analysis Memo # 8-71

TO: Distribution

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SUBJECT: Concise Analytic Approximations to the Sun's Geocentric Position Vector.

SUMMARY

The solar geocentric position vector is the negative or inverse of the earth's heliocentric position vector. While the famous JPL Ephemeris Tapes are the ultimate in accurate earth (and planetary) ephemerides, for many applications they require too much computer storage and data handling and are far more accurate than is really necessary for the purpose at hand.

This memo presents the classical analytic conic approximation to the solar geocentric position vector, carried through fourth order in the earth's eccentricity. The user may easily truncate the given expressions to third, second or even first order, thereby considerably simplifying an already fairly simple formulation. (The first order expressions yield a solar position angular accuracy of about 1.5 arc minutes over a period of several years.) This memo also presents the explicit formulas for calculating the four mean-element "constants", which are required in the solar position approximation expressions, with respect to any mean equator and equinox coordinate system during the 20th Century. Special formulas are given for the widely used 1950.0 coordinate system.

DETERMINATION OF THE MEAN ELEMENT "CONSTANTS"

The mean elements of the orbit of the earth-moon barycenter around the sun are very slowly varying over a century, with the single exception of the mean anomaly which increases linearly by approximately 360° a year as well as having a very slowly varying component. Hence for any time period of several years, these mean elements may be approximated by constants except for the mean anomaly which may approximated by a constant plus a linear term.

This section describes how the mean element constants may be calculated for various time-points and in various coordinate systems. The method used, and hence the actual values obtained, will determine in which coordinate system the solar position, which is computed from the general formulae given in the next section, will be expressed:

Let the following times be defined:

t : time-point in Julian Ephemeris Days at which the solar position is desired;

t_0 : time-point in Julian Ephemeris Days which will be taken as the epoch in the solar position approximation. [i.e., the approximation will be expressed entirely in terms of the time interval $(t - t_0)$ and the mean-element constants].*

t_c : time-point in Julian Ephemeris Days at which the constants approximating the mean elements are precisely correct. [i.e., the time-point to which the slowly varying mean elements are updated by the formulae of this section].

The mean element constants are computed directly from the expressions given in one of the following sub-sections according to the desired coordinate system.

Mean-of-Date Coordinate System

This coordinate system is the one defined by the instantaneous mean equator and equinox of the time-point t_c to which the mean elements are updated. That is, the updating expressions given below already contain the corrections for precessing the coordinate system to the time-point t_c** as well as the terms for updating the mean elements to the same time-point t_c. In practice, since the coordinate system is precessing faster (about 50 arc-seconds per year) than the mean elements are changing (the largest rate of change, other than the linear term in the mean anomaly which is accounted for explicitly, being about 12 arc-seconds per year in the longitude of perihelion), the time-point t_c should be chosen so as to obtain the

*See the last section of this memo for the choice of t_0 for the various Apollo Nearest Besselian Year Coordinate Systems.

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coordinate system desired rather than as a mid-point of the time interval over which the solar position approximation is desired. Thus, it is tacitly assumed that one does not want the solar position at a time more than a few years away from the time-point associated with the coordinate system. Consequently if the solar position is desired in the 1970's with respect to a 1950.0 coordinate system, one must use other expressions (namely, those in the 1950.0 coordinate system subsection of this memo).

Let

$$D = (t_c - 2415020.0)/10^4.$$

The number 2415020.0 is the Julian Ephemeris Date corresponding to January 0.5, 1900.

The mean obliquity of the ecliptic at the time t_c is:

$$\epsilon = 23.452294^\circ - 3.5626(10)^{-3} D - 1.23(10)^{-7} D^2 + 1.03(10)^{-8} D^3$$

The mean eccentricity of the earth-moon barycenter at the time t_c is:

$$e_{EM} = 0.01675104 - 1.1444 \ (10)^{-5} D - 9.4(10)^{-9} D^2.$$

The longitude of perihelion of the earth-moon barycenter at the time t_c is:

$$\tilde{\omega}_{EM} = 101.220833^\circ + 0.470684 D + 3.39(10)^{-5} D^2 + 7.0(10)^{-8} D^3.$$

The mean anomaly of the earth-moon barycenter fully updated to the time t_c but linearly backdated to the epoch t_0 is:

$$M_{0EM} = 358.475845^\circ + n_{EM}(t_0 - 2415020.0) - 1.12(10)^{-5} D^2 - 7.0(10)^{-8} D^3$$

where the mean motion n_{EM} of the earth-moon barycenter is:

$$n_{EM} = 0.9856002670 \text{ degrees/ ephemeris day}$$

The above expressions are given in Melbourne et al. (1968, pg. 30).

1950.0 Coordinate System

This coordinate system is the one defined by the instantaneous mean equator and equinox of the begining of the 1950 Besselian Solar Year which corresponds to the Julian Ephemeris Date 2433282.423357. The JPL Ephemeris Tapes use this coordinate system, and it has been adopted as the Skylab Basic Reference Coordinate System. The expressions given below update the mean elements to the time-point t_c but express the result in the 1950.0 coordinate system; no precessional corrections are contained in these expressions. The time-point t_c should be selected as the mid-point of the time interval over which the solar position approximation is desired so that the mean element constants obtained will be very nearly the average values of the elements over the time interval.

Let

$$T = (t_c - 2433282.5)/36525 .$$

The number 2433282.5 is the Julian Ephemeris Date corresponding to January 1.0, 1950. This differs slightly from the Julian Ephemeris Date defining the coordinate system, but the above value is less awkward and is used in Sturms (1971), from which the following expressions are taken.

The mean obliquity of the ecliptic at the time t_c is:

$$\epsilon = 23.4457888616^{\circ} - 0.0130141669 \, T - 9.445(10)^{-7} T^2 + 5.0(10)^{-7} T^3 .$$

The mean eccentricity of the earth-moon barycenter at the time t_c is:

$$e_{EM} = 0.0167301085 - 4.1926(10)^{-5} T - 1.26(10)^{-7} T^2$$

The longitude of perihelion of the earth-moon barycenter at the time t_c is:

$$\tilde{\omega}_{EM} = 102.08053^{\circ} + 0.32328\ T + 1.5(10)^{-4}\ T^2$$

The mean anomaly of the earth-moon barycenter fully updated to the time t_c but linearly backdated to the epoch t_0 is:

$$M_{0EM} = 358.000682^{\circ} + n_{EM}(t_0 - 2433282.5) - 1.55(10)^{-4}\ T^2 - 3.3333(10)^{-6}\ T^3$$

where the mean motion n_{EM} of the earth-moon barycenter is:

$$n_{EM} = 0.9856002628 \text{ degrees/ ephemeris day}$$

DETERMINATION OF THE SOLAR POSITION

The following sequence of computations is performed for the solar position approximation, using the mean element constants (ϵ, e_{EM}, $\tilde{\omega}_{EM}$, M_{0EM}, n_{EM}) determined by one of the methods of the previous section.

The mean anomaly of the earth-moon barycenter at the arbitrary desired time t is:

$$M_{EM} = M_{0EM} + n_{EM}(t - t_0)$$

The true anomaly of the earth-moon barycenter at the arbitrary desired time t is:*

$$f_{EM} = M_{EM} + \left(2e_{EM} - \frac{1}{4}e_{EM}^3\right)\sin M_{EM} + \left(\frac{5}{4}e_{EM}^2 - \frac{11}{24}e_{EM}^4\right)\sin 2M_{EM} + \frac{13}{12}e_{EM}^3 \sin 3M_{EM} + \frac{103}{96}e_{EM}^4 \sin 4M_{EM}$$

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*This equation is given in Battin (1964, problem 2.23), and a more general equation correct through terms of e_{EM}^7 is given in Stumpff (1959, page 314).

This equation is correct through terms of e_{EM}^4. However, since e_{EM} is small the fourth, third, and sometimes second order terms may be neglected depending on the desired trade-off between accuracy and computing speed. The maximum error introduced by neglecting all but the first order term is about 1.5 minutes of arc.

The true longitude of the earth-moon barycenter at the arbitrary desired time t is:

$$L_{EM} = f_{EM} + \tilde{\omega}_{EM}$$

The true longitude of the sun at the arbitrary desired time t relative to the earth-moon barycenter is hence:

$$L_{SUN} = L_{EM} - 180^{\circ}$$

The solar position unit vector relative to the earth-moon barycenter is then:

$$\underline{1}_{SUN} = (\cos L_{SUN},\ \cos\epsilon\ \sin L_{SUN},\ \sin\epsilon\ \sin L_{SUN})$$

The solar distance relative to the earth-moon barycenter at the arbitrary desired time t is:

$$r = a_{EM} \Big\{ 1 + e_{EM}^2/2 - (e_{EM} - \frac{3}{8} e_{EM}^3) \cos M_{EM} - (\frac{e_{EM}^2}{2} - \frac{e_{EM}^4}{3}) \cos 2M_{EM} - \frac{3}{8} e_{EM}^3 \cos 3M_{EM} - \frac{1}{3} e_{EM}^4 \cos 4M_{EM} \Big\}$$

where

$$a_{EM} = 1.49597927\,(10)^8 \text{ km}$$

This equation is correct through terms of e_{EM}^{4*}. Again, fourth, third and perhaps second order terms may be neglected depending on the accuracy desired.

The declination δ and right ascension α of the sun at the arbitrary desired time t are:

$$\delta_{SUN} = \arcsin(\sin\epsilon \sin L_{SUN})$$

$$\alpha_{SUN} = S_\alpha \arcsin(\tan\delta_{SUN}/\tan\epsilon) + 90^\circ(1-S_\alpha) + 90^\circ(1+S_\alpha)(1-S_\delta)$$

where

$$S_\alpha = \text{sign}(\cos L_{SUN})$$

$$S_\delta = \text{sign}(\sin L_{SUN}) = \text{sign}(\delta_{SUN})$$

These expressions are obtained by the solution of spherical triangles, and are formulated so that $-90^\circ \leq \delta_{SUN} \leq +90^\circ$ and $0^\circ \leq \alpha_{SUN} < 360^\circ$.

The solar position unit vector relative to the earth-moon barycenter may also be expressed as:

$$\underline{1}_{SUN} = (\cos\alpha_{SUN}\cos\delta_{SUN},\ \sin\alpha_{SUN}\cos\delta_{SUN},\ \sin\delta_{SUN})$$

APPLICATION: ANALYTIC COMPUTATION OF "LOSSEM" SOLAR CONSTANTS

The Solar Ephemerides Subroutine in the Lunar Module Guidance Computer has always used the solar position unit vector approximation:

$$\underline{1}_{SUN} = (\cos(LOS),\ K_1 \sin(LOS),\ K_3 \sin(LOS))$$

where

$$LOS = LOS_0 + LOS_R(t - t_0) - C\sin(OMEGA_C(t - t_0) + PHASE_C)$$

*A more general equation correct through terms of e_{EM}^{7} is given in Stumpff (1959, pg. 312).

The constants LOS_0, LOS_R, C, $OMEGA_C$ and $PHASE_C$ have traditionally been determined empirically by curve-fitting data from the JPL Ephemeris Tapes over specified time intervals. See McOuat (1965, 1969, 1970a, 1970b), Canepa (1970), White (1970), and Rich (1970a, 1970b).

However, by comparison with the expressions in the previous section it may be seen that the five constants may be approximated by the relations:

$$\begin{aligned} LOS_0 &= \tilde{\omega}_{EM} - 180^{\circ} + M_{0EM} \\ LOS_R &= n_{EM} \\ C &= -\left(2e_{EM} - \frac{1}{4}e_{EM}^3\right) \\ OMEGA_C &= n_{EM} \\ PHASE_C &= M_{0EM} \end{aligned}$$

or

$$\begin{aligned} LOS_0 &= \tilde{\omega}_{EM} + M_{0EM} - 180^{\circ} \\ LOS_R &= n_{EM} \\ C &= +\left(2e_{EM} - \frac{1}{4}e_{EM}^3\right) \\ OMEGA_C &= n_{EM} \\ PHASE_C &= M_{0EM} - 180^{\circ} \end{aligned}$$

and the obliquity constants are given by:

$$\begin{aligned} K_1 &= \cos \epsilon \\ K_3 &= \sin \epsilon \end{aligned}$$

These relations provide a fast and convenient computation technique for the LOSSEM constants for any coordinate system and epoch.

It must be mentioned that the empirical determination of these constants by curve-fitting can yield slightly more accurate results in the solar position unit vector over specified time intervals. However, the accuracy improvement is generally negligible: the angular difference between the solar position vectors as calculated from curve-fit constants and from analytic constants is generally less than 9 arc-seconds (0.0025°), while the maximum angular difference between the true (JPL) solar position vector and either approximation is about an order of magnitude larger. As one would expect, the latter difference (true minus either approximation) varies more or less sinusoidally with a period of about 182 days, and comes from the neglect of the $\sin 2M_{EM}$ term in the true anomaly expression.

CHOICE OF TIME-POINTS (t_0 and t_c) FOR THE VARIOUS APOLLO NEAREST BESSELIAN EPHEMERIS YEARS AND THEIR ASSOCIATED COORDINATE SYSTEMS

In order to be consistent with the epochs and the coordinate systems of the various APOLLO Nearest Besselian Ephemeris Years, the time-points (t_0 and t_c) should be selected as indicated below.

Choice of the time-point t_c

For the Apollo Nearest Besselian Year Coordinate System choose t_c as follows*:

NBY 1969/1970 (Apollo 11, 12, 13): t_c = 2440587.2672387 E.T. = 1970.0 E.T.
NBY 1970/1971 (Apollo 14) : t_c = 2440952.5094319 E.T. = 1971.0 E.T.
NBY 1971/1972 (Apollo 15, 16, 17): t_c = 2441317.7516251 E.T. = 1972.0 E.T.
NBY 1972/1973 (-------------): t_c = 2441682.9938182 E.T. = 1973.0 E.T.

These values are the Julian Ephemeris Dates of the beginnings of the respective Besselian Years, and determine the reference time of the coordinate system.

Choice of the time-point t_0

In the Apollo Guidance Computers, the epoch t_0 is always chosen at the July 1.0 universal time which is (approximately) the beginning of the Nearest Besselian Year in question (usually the NBY during which the launch occurs). In this memo, ephemeris time has been used. However, since the arbitrary time t at which the solar position approximation is desired is also measured in universal time in the AGC's, and since only the difference $t - t_0$ is ever used in the approximation equations and then always as a multiplier of the mean motion n_{EM}, we may utilize the approximation equations in universal time provided only that we convert the mean motion to an angular rate in universal time. This is accomplished by multiplying the previous values of n_{EM} by the factor $(1 + 2.852(10)^{-8})$, which

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*For the Skylab 1950.0 coordinate system, t_c should be chosen as the midpoint of the time interval over which the solar position approximation is desired, as indicated in that section of this memo.

expresses the fact that universal time is slowing down with respect to ephemeris time by 0.9 second per year. It is not necessary to know the absolute difference ΔT between ephemeris and universal time at any time-point. When evaluating the mean element constants(ϵ, e_{EM}, $\tilde{\omega}_{EM}$, M_{0EM}, n_{EM}) however, t_0 must be set to the desired epoch expressed in ephemeris time, such as one of the following:

NBY 1969/1970 (Apollo 11, 12, 13) : t_0 = 2440403.5 = July 1.0, 1969, E. T.
NBY 1970/1971 (Apollo 14) : t_0 = 2440768.5 = July 1.0, 1970, E. T.
NBY 1971/1972 (Apollo 15, 16, 17) : t_0 = 2441133.5 = July 1.0, 1971, E. T.
NBY 1972/1973 (--------) : t_0 = 2441499.5 = July 1.0, 1972, E. T.

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